

A STUDY OF THE EFFECT OF INTERNAL FRICTION
ON RUPTURE MODULUS OF WOOD

By

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ABSTRACT

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The major emphasis in this study was to determine any statistical correlation between the internal friction, as determined by a one-dimensional longitudinal stress-wave technique, and the rupture modulus of wood. The secondary objective of this study was to investigate the relationship between Young's modulus of wood as determined by compressive and tensile stress waves. Twenty-six 1/2 by 1/2-inch cross section and 42-inch long Douglas fir (*Pseudotsuga menziesii*) clear wood specimens were first tested under impact loading conditions to measure the internal friction and dynamic Young's modulus. The specimens were prepared in such a manner as to contain grain deviations of zero, 1/15, and 1/10. These specimens were later tested under a conventional static bending test to determine Young's modulus and rupture modulus of wood.

From the results of this study, it appears that no useful relationship exists between internal friction and rupture modulus of wood. Dynamic Young's modulus, however, displays a very good correlation coefficient of 0.962 with static Young's modulus. Also, it was found that dynamic compressive Young's modulus in the longitudinal direction was about 1.9% higher than dynamic tensile Young's modulus for wood. This result, for an anisotropic material such as wood, agrees well with the situation known to exist with isotropic materials.

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INTRODUCTION

Wood is one of the most versatile and widely used construction materials. Yet there are relatively inaccurate methods of determining its limiting strength properties. More often than not, the stress grading of wood is done on the basis of limiting strength of the weakest piece to be found in the grade. Furthermore, strength determinations are carried out on a sample basis and the results then projected to the larger population by visual techniques. This involves approximations, which results in a wide spectrum of strength in any grade. The ideal grading system for wood would be the one in which the spectrum is greatly restricted and no pieces below the stated value are present in the grade. This can be approached only through non-destructive testing techniques. The main objective of the present work is related to this goal.

The problems involved in developing more reliable and efficient techniques for predicting wood strength are due to the inherently complex wood structure itself. Wood, being a biological material, involves many variables that are virtually impossible to control for purposes of laboratory testing.

Wood by nature is an anisotropic material. It also shows heterogeneity at a much higher macroscopic level than many other materials. But if the local heterogeneities and the curvature of the yearly growth rings are neglected, wood exhibits three orthogonal planes of structure symmetry and thus falls in the special category of anisotropic materials known as orthotropic materials.

Wood properties vary not only from specimen to specimen but can also vary within the same specimen, making exact duplication of the experiments impossible. Nevertheless, reliable general trends can be established based upon statistical inferences.

With the problems of complexity in the background, the primary objective of this study was to determine any useful statistical correlation which may exist, between the strength properties of wood and its internal friction. This correlation alone, or in conjunction with some other parameters, if significant, can then be used as a basic criterion for non-destructive wood testing and grading.

The secondary objective of this study was to study the Young's modulus of wood in compression and tension. For an orthotropic material like wood, as will be shown later in this work, as many as nine independent material constants are required to relate strains and stresses at any point in the body for any general case of loading. Of these nine constants, three are the Young's moduli, defined in the three orthogonal directions of elastic symmetry of the orthotropic material. In contrast, isotropic materials have only two independent material constants. It has been known that the Young's modulus for isotropic materials differ for compressive and tensile loadings such that Young's modulus in compression is about 2% higher than that in tension. It was considered important, then, to determine if such was the case for Young's modulus of wood in longitudinal direction.

The general strategy to meet these objectives was to test Douglas fir clear wood specimens of different density levels and carefully controlled grain slopes. It is well established that the grain slope has a very pronounced effect on rupture modulus. Therefore, specimens of different grain slopes were selected to get a wide spread in rupture modulus data. Under the

conditions of impact loading, the attenuation and stress pulse velocity would be determined to calculate internal friction and dynamic Young's modulus in compression and in tension. In the second phase of testing, these specimens would be tested to failure under a static bending condition to determine the rupture modulus. All data would then be reduced statistically to establish any correlation between the measured quantities.

Many investigators have successfully attempted to evaluate elastic properties of wood using the relatively new techniques of piezoelectric effect, vibrational properties, and dynamic response under impulse loading conditions of wood. Mohr (13) showed that wood exhibits a pronounced increase in the elastic modulus of compression in radial and tangential orientations for oscillatory type of loading. Jayne (10) showed a relationship between the attenuation of mechanical transverse vibrations and the rupture modulus of wood. He used 0.75 by 1.0 by 8.0-inch Sitka spruce wood specimens and determined the damping capacity from the resonance frequency amplitude response. He found that the correlation between the strength properties of wood, determined from static bending tests, and density of wood is improved if the density is combined with the damping capacity, from 0.267 to 0.824. Pellerin (17) used 2-inch-thick wood boards of varying lengths from 12 to 16 feet and 6 to 12 inches wide under free vibrations. It was found that the correlation between log decrement of the transverse vibrations and the rupture modulus, as determined from the static bending tests, was generally improved if material properties like Young's modulus were combined with the log decrement of the vibrations. However, James (8), using longitudinal acoustic waves, found no significant relation between attenuation and rupture modulus, contradicting the results of Jayne and Pellerin.

Another way of measuring the dissipation of elastic energy in solids, due to the phenomenon called internal friction, is to determine the attenuation of a propagating stress pulse in the material produced under impact loading conditions. The theory governing the stress-wave propagation in solids is rather complex for generalized cases. In fact, no exact mathematical solutions are yet available, even for isotropic materials, except for the very simplest cases (11). Fortunately, approximate solutions for longitudinal stress waves propagating in long bars of any uniform cross section are available. These can be applied with relative ease to determine the dynamic response of long bars.

In comparison, much less work has been done to analyze the stress-wave propagation behavior for an anisotropic medium. No doubt this is because of the apparent complexity of problems for anisotropic materials involving up to 21 independent elastic constants instead of the conventional two for isotropic materials. Approximate solutions for anisotropic bars of rectangular cross section were obtained by Nigro and Nigro and Andrews for longitudinal and flexural waves (14,15). Nowinski (16) obtained approximate solutions for infinite cylindrical orthotropic bars. All these mathematical solutions tend to be too involved for application in practice.

Bertholf (2) tested strain pulse propagation in wood bars generated by the impact of two bars traveling at different velocities. He showed that the one-dimensional elastic wave propagation theory is adequate for the prediction of the major elements of strain waves in wood rods. Bhalla (3) tested cylindrical Douglas fir wood bars under impact loading conditions. He found that the pulse propagates in wood without significant dispersion but decayed more rapidly than in metal bars. He also showed that the shape, amplitude, and velocity of the axial strain pulse remained the same in the

tangential and radial planes. However, dynamic Poisson's ratio was higher by 19% in the tangential plane than in the radial plane.

THEORY

Stress-Wave Propagation in Rods

In the theory of elasticity, it is assumed that the body is in equilibrium under the action of applied external forces and that its elastic deformations have reached their static values. However, when considering the effects of forces applied for very short periods or forces which are rapidly changing, the inertia effects must be considered in terms of stress-wave propagation. The theory of stress waves in solids predicts that in the interior of an extended isotropic medium, two types of elastic waves can propagate. These are known as waves of dilatation, which travel with velocity $(\lambda + 2\mu/\rho)^{\frac{1}{2}}$, and waves of distortion, which travel with the velocity $(\mu/\rho)^{\frac{1}{2}}$. The material constants λ and μ are Lamé's constants as defined in the theory of elasticity, and ρ is the mass density of the medium.

The propagation of stress waves in any bounded isotropic solid can be studied by solving the equations of motion of an isotropic elastic solid for the appropriate boundary conditions. These solutions, however, excepting the very simplest cases, tend to be very complex and require a numerical approach. The complexity of the solution reduces in case of thin bars where types of vibrations that can propagate may be classified as longitudinal, torsional and lateral.

In longitudinal vibrations, the particle motion is considered to be in only one direction, i.e., in the axial direction. Thus, under longitudinal vibrations, bar elements elongate or contract with the lateral motion neglected. In torsional vibrations, plane transverse sections rotate about

their center; and in lateral vibrations, elements of the central axis move laterally.

Consider an element LM of a rod of uniform cross-sectional area A (Fig. 1). If the longitudinal stress pulse passing the element has a magnitude σ_x at plane L, the stress at section M can be written, using a first-order Taylor series expansion, as:

$$\sigma_x + \frac{\partial \sigma_x}{\partial x} dx$$

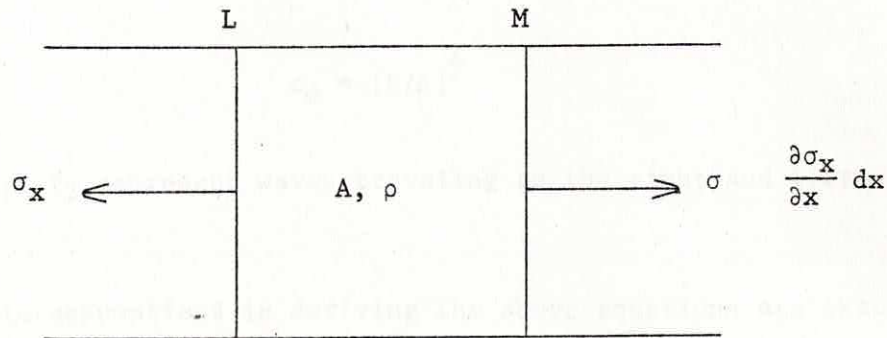


Fig. 1.--Stresses on an element of bar in longitudinal motion.

Applying Newton's second law of motion, the following differential equation is obtained:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_x}{\partial x} \quad [1]$$

where ρ is the mass density of the rod material, u is the axial displacement, t is time, and x is the axial coordinate. Also, since Hooke's Law in one dimension for an isotropic material is:

$$\sigma_x = E \frac{\partial u}{\partial x} ,$$

Eq. [1] can be written:

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} \quad [2]$$

Equation [2] is the equation of longitudinal motion of rod material on the one-dimensional wave equation. The solution can be represented in the functional form:

$$u = f_1(c_0 t - x) + f_2(c_0 t + x) \quad [3]$$

where c_0 is the rod wave propagation velocity and is given as:

$$c_0 = [E/\rho]^{1/2} \quad [4]$$

The functions f_1 , f_2 represent waves traveling to the right and left, respectively.

The basic assumptions in deriving the above equations are that the plane transverse sections remain plane during the propagation of a stress wave through them and that the stress acts uniformly over the plane section. This assumption implies that there is no other motion of the particle except longitudinal. But in actual practice, however, a longitudinal strain as a result of a longitudinal stress is always necessarily accompanied by a Poisson's or lateral strain. This will give rise to a radial motion of the particle in addition to its longitudinal motion. This in turn will result in an uneven stress distribution across the sections. However, the effects of this non-uniform stress distribution is of importance only when the operative wavelength of the stress pulse is of the same order as the lateral dimensions of the rod (11).

It should therefore be noted that, in view of the previous assumptions, the above solution is only an approximate solution. Also, it must be emphasized that no assumption regarding the shape of the cross section is made except that the rod is of uniform cross section. Thus, the above derived equations shall apply to all thin rods of any uniform cross sections (11).

Internal Friction

The theory of stress-wave propagation derived in the previous section is based on Hooke's Law. The equation of motion is obtained by equating elastic forces of the solid medium to the product of its mass and its particle acceleration according to Newton's second law of motion. It has been assumed that no other forces come into play. In actual practice, however, a stress pulse propagating through a real material is always attenuated and ultimately dies out even if the system is perfectly isolated and there is no energy transfer to its surroundings. Some of the elastic energy is invariably converted into heat. The exact mechanisms which result in this phenomenon are rather complex and have not yet been analyzed mathematically (11).

Internal friction can be defined, without making any reference to its nature, by the ratio $\Delta W/W$ where ΔW is the energy dissipated in taking a specimen through a stress cycle and W is the elastic energy stored in the specimen when the strain is a maximum. This ratio is also sometimes referred to as "specific damping capacity" or "specific loss."

One of the several other indirect methods of defining internal friction is called the logarithmic decrement. It is based upon the assumption that the restoring forces are proportional to the amplitude of the vibrations, whereas the dissipative forces are proportional to the velocity of the vibrations. Under these conditions, the natural logarithm of the ratio of the

successive oscillations is constant. It can be proven that the logarithmic decrement " δ " is one-half of the specific loss (11).

The attenuation for a plane sinusoidal wave of small amplitude is found to be exponential so that a stress pulse of original amplitude, σ_0 , will have an amplitude:

$$\sigma = \sigma_0 e^{-\alpha x} \quad [5]$$

where σ is the stress amplitude after traveling through a distance x . The symbol α is called the attenuation constant and is found from the Eq. 5 to be:

$$\alpha = \frac{1}{x} \ln \frac{\sigma_0}{\sigma} \quad [6]$$

Consider a slab of unit cross-section area and thickness Δx of a given material. The energy flux for a plane wave of stress amplitude σ_0 is given by:

$$f = \frac{\sigma_0^2}{2\rho c_0} \quad [7]$$

Then the energy entering per unit time for a wave normal to the cross section is given as:

$$\dot{w}_1 = \frac{\sigma_0^2}{2\rho c_0} e^{-2\alpha x} \quad [8]$$

whereas the energy leaving per unit time will be:

$$\dot{w}_0 = \frac{\sigma_0^2}{2\rho c_0} \cdot e^{-2\alpha(x + \Delta x)} \quad [9]$$

as shown in Fig. 2.

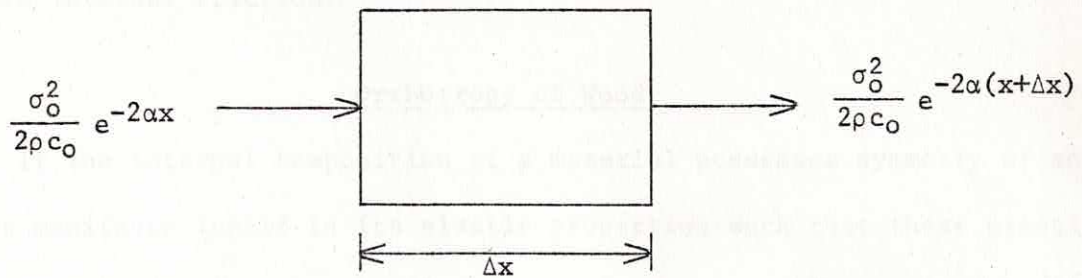


Fig. 2.--Energy flux entering and leaving an element.

Thus the energy dissipated per second in the slab is approximately:

$$\dot{W} = \frac{\sigma_0^2 \alpha \Delta x}{\rho c_0} \cdot e^{-2\alpha x} \quad [10]$$

If f is the frequency of oscillations of a stress wave, then the energy dissipated per cycle, ΔW , is given as:

$$\Delta W = \frac{\sigma_0^2 \alpha \Delta x}{f \rho c_0} \cdot e^{-2\alpha x} \quad [11]$$

Also, the energy density in the medium can be written:

$$\frac{\sigma_0^2}{2\rho c_0^2}$$

Hence, the maximum energy stored in the slab becomes:

$$W = \frac{\sigma_0^2 \Delta x}{2\rho c_0^2} \cdot e^{-2\alpha x} \quad [12]$$

From Eqs. [11] and [12], we get:

$$\frac{\Delta W}{W} = \frac{2\alpha c_0}{f} \quad [13]$$

Equation [13] in conjunction with Eq. [6] is used exclusively in this study to calculate internal friction.

Orthotropy of Wood

If the internal composition of a material possesses symmetry of any kind, it manifests itself in its elastic properties such that these elastic properties are identical in the directions of symmetry. This principle, set forth by F. Neumann for construction symmetry in crystalline materials, can also be extended to structure symmetry in wood, plywood, etc. (12).

Wood exhibits three orthogonal planes of structure symmetry, and these are also its planes of elastic symmetry. Materials that exhibit three planes of elastic symmetry are called orthotropic materials. It is shown in Appendix A that for such a material, nine independent elastic constants are required to relate stresses and strains at any point in the body for any general case of loading.

PROCEDURE

Specimen Preparation

A total of 26, 42-inch-long, $1/2$ by $1/2$ -inch cross-section straight bar specimens of Douglas fir clear wood were prepared. Density and grain angle of wood were to be controlled parameters so that the specimens were cut at three different grain angles from three boards of distinct density levels. These boards had been stored in dry conditions for a period of about three years. This ensured that the moisture content had reached an equilibrium condition. Cuts were made parallel to the wood grain and at a slope of $1/15$ and $1/10$. It is well known that temperature and moisture content influence rupture modulus of wood. In this study, however, these two variables were monitored and kept at ambient conditions.

Each specimen was accurately weighed and its density determined after lightly sanding all surfaces with a fine grade sandpaper. Budd type C-12-121 foil strain gages were mounted at two stations on the wood bar, each station located 9 inches away from the ends of the bar. At each station, two strain gages connected in series were mounted on the opposite tangential planes and oriented in the longitudinal direction of the bar. The series connections of the strain gages were done to cancel any stresses due to bending. Eastman 910 adhesive was used to bond the strain gages to the wood surface. A satisfactory performance of strain gages bonded on wood has previously been reported (18,20).

A 42-inch-long, $1/2$ by $1/2$ -inch cross-section aluminum loading bar was then bonded to one end of the wood bar specimen using an epoxy adhesive. The

twofold advantage of indirectly loading the wood specimen through the loading bar is explained in a later section. A schematic sketch of a finished specimen with the aluminum loading bar attached is shown in Fig. 3.

Experimental Apparatus

To induce a stress pulse in the specimen, a 1/2-inch diameter chrome steel ball was impacted onto the end of the loading bar using a high-pressure gas gun having a 1/2-inch diameter barrel. Although the gun is capable of projectile velocities on the order of 1500 ft./sec., impact velocities in the present study were limited to on the order of 30 ft./sec.

A seven-channel potentiometer circuit with a built-in strain gage calibration feature was employed to obtain a voltage signal from the strain gages. The signals from the strain gages were recorded on a Tektronix model 7503 oscilloscope equipped with a 7A12 dual channel vertical amplifier. The amplifier had a maximum sensitivity of 5 mV/div. for each channel and a frequency response of 75 MHz. To get a permanent record of the oscilloscope trace, a Tektronix C-50 oscilloscope camera with Polaroid Type 107 film was used.

As an external triggering source for the oscilloscope trace, an Endevco P.R. accelerometer with a Kistler model S/N 1145 charge amplifier was used. A Nikon Measurescope equipped with X-Y traveling stage and X30 magnification was employed for strain gage and strain gage calibration data reduction from Polaroid print.

The rupture modulus of the wood specimens was determined using a standard third-point loading of beams technique. The loading to rupture of the specimens was carried out on an Instron testing machine, which had a 0-20,000 lbs. maximum full-scale loading capacity. A 0-200 lbs. maximum

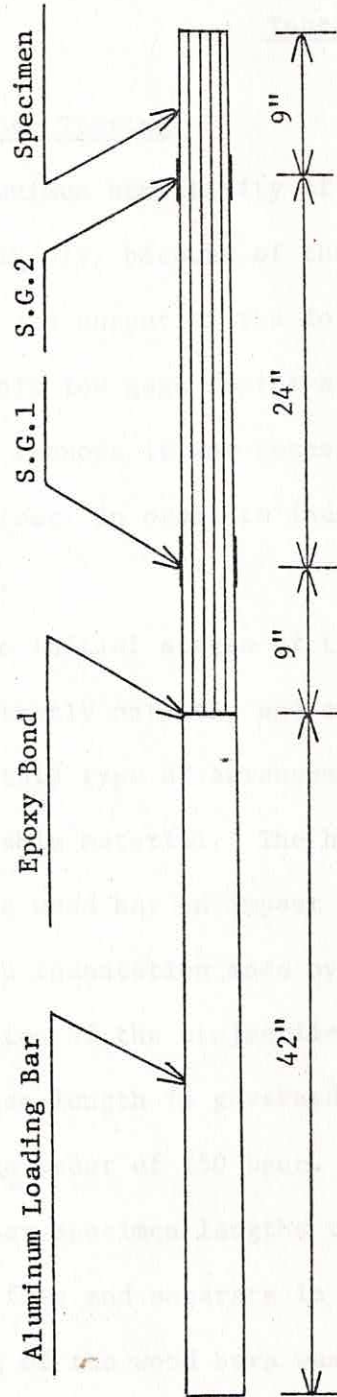


Fig. 3.--Schematic sketch of a finished specimen.

full-scale range of the machine was used to load the specimens in the present work.

Testing Procedure

Dynamic Response Testing

The maximum sensitivity of the oscilloscope that could be used was limited to 5 mV/div. because of the noise pick-up problems at higher amplifier gains. Also, the output of the foil type strain gages is comparatively low because of their low gage factor as compared to semiconductor strain gages. For these two reasons it was necessary to use projectile impact velocities of around 30 ft./sec. in order to induce a significant strain amplitude in the wood specimen.

In the initial stages of the study, trials were made by impacting the projectile directly onto the end of a wood bar. There were two problems involved for this type of arrangement. Wood is comparatively soft and an easily indentable material. The high velocity projectile caused damage to the end of the wood bar on impact due to crushing and splintering of the wood. Also, the deep indentation made by the projectile into the wood caused the contact duration of the projectile and wood bar to be comparatively long. Since the pulse length is governed by the contact duration, stress pulses of lengths on the order of 150 μ sec. were obtained. This would necessitate much longer wood bar specimen lengths to keep the original and reflected stress pulse at the free end separate in the strain data. To overcome these problems, loading of the wood bars was done through an aluminum loading bar arrangement. The much harder aluminum bar protected the end of the wood bar from any damage and also resulted in a pulse length of about 50 μ sec.

It was observed that the velocity of pulse propagation in wood and aluminum is nearly the same, about 0.2×10^6 in./sec. A compressive pulse, σ , is initially generated in the aluminum loading bar by the longitudinal impact of a projectile. A part of this pulse, $(\sigma_1)_T$, is transmitted through the aluminum-wood interface into the wood bar and part of it, $(\sigma_2)_R$, is reflected back in tension into the aluminum bar (Fig. 4). The time required

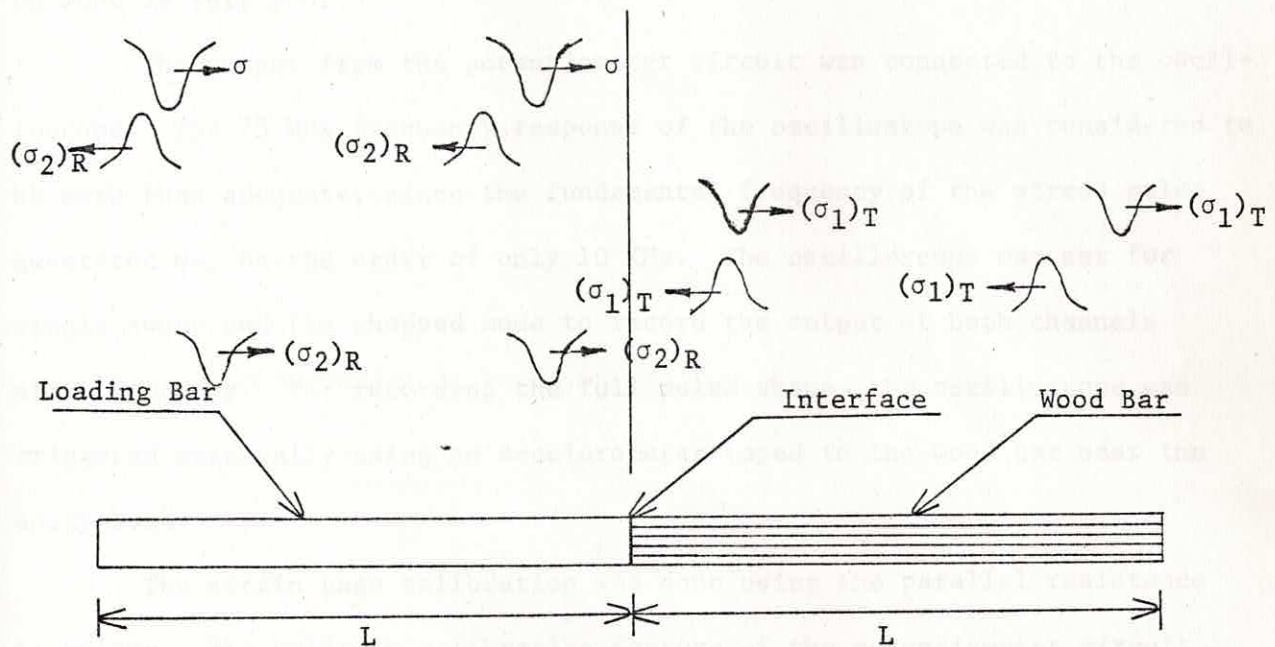


Fig. 4.--Stress-pulse propagation in the loading bar and specimen wood bar.

for the transmitted pulse in wood to travel through its length, reflect back in tension from the free end and reach the interface again would be given as $2L/c_0$. If the velocity of the pulse propagation is assumed to be equal for both the materials, then the minimum length of the loading bar must also be equal to L to ensure no superposition of the two pulses in the strain data. In preparation for testing, the specimen axis was supported and aligned co-linear with the gun barrel. This was to ensure the least amount of bending

under dynamic loading. Also, to minimize the amount of elastic energy transferred to the supports, the specimens were supported on foam rubber, placed in an angle section.

The leads from the strain gages were then connected to the potentiometer circuit. A 610 ohm ballast resistance was selected to limit the current to less than 20 mA in the strain gages to avoid overheating. This was an important consideration, since heat dissipation from strain gages mounted on wood is very poor.

The output from the potentiometer circuit was connected to the oscilloscope. The 75 MHz frequency response of the oscilloscope was considered to be more than adequate, since the fundamental frequency of the stress pulse generated was on the order of only 10 KHz. The oscilloscope was set for single sweep and the chopped mode to record the output of both channels simultaneously. For recording the full pulse shape, the oscilloscope was triggered externally using an accelerometer taped to the wood bar near the epoxy bond.

The strain gage calibration was done using the parallel resistance technique. The built-in calibration feature of the potentiometer circuit connects and disconnects a shunt resistance R_{cal} , using a 60 cps oscillator as shown in Fig. 5. The voltage change due to connecting and disconnecting

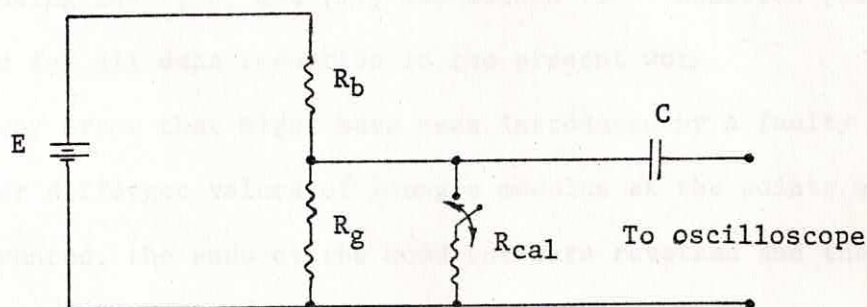


Fig. 5.--Potentiometer circuit with parallel resistance calibration technique.

the parallel resistance can be interpreted as an equivalent strain according to the following equation:

$$\epsilon_{eq} = - \frac{R_g}{F(R_g + R_{cal})} \quad [14]$$

where R_g is gage resistance and F the gage factor. A calibration constant can be determined from:

$$k = \frac{\epsilon_{eq}}{\Delta E_{gcal}} \quad [15]$$

Then, the actual strain, ϵ , is given as:

$$\epsilon = k \Delta E_g \quad [16]$$

where ΔE_{gcal} is the voltage change due to the shunt resistance and ΔE_g is the strain gage output voltage..

The strain, ϵ , can also be calculated from the circuit equation:

$$\epsilon = \frac{(R_g + R_b)^2}{R_g R_b F E} \cdot \Delta E_g \quad [17]$$

where R_b is the ballast resistance (5). Agreement between the values of strains calculated using Eqs. [16] and [17] was within .5%. Equation [16] was exclusively used for all data reduction in the present work.

To correct any error that might have been introduced by a faulty strain gage output or different values of Young's modulus at the points where strain gages were mounted, the ends of the wood bar were reversed and the second end was then bonded with the loading bar. The whole procedure was

repeated. The averaged data from both runs were then used in the analysis (Appendix B).

Static Testing

The second phase of the study consisted of testing the specimens for their modulus of rupture. A standard third-point loading of beams technique was employed (1). Specimens were tested on an Instron testing machine. The machine was calibrated prior to testing the specimens by putting known dead weights on its load cell. An 18-inch span length of the beam was selected (Fig. 6). The dynamic response of the wood bars was essentially an average

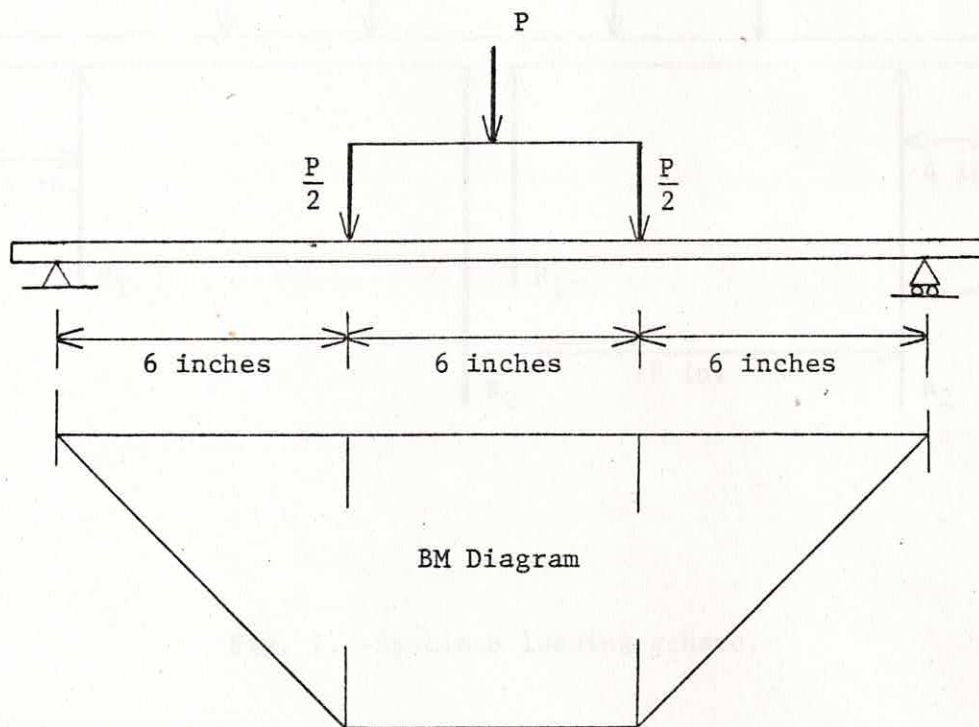


Fig. 6.--Third-point loading of beams.

over the 24-inch bar length between the strain gage stations. Therefore, in order to get average rupture strength of the 24-inch bar length between the strain gage stations, the loadings of the specimens were done according to the loading scheme shown in Fig. 7, where subscript 1 refers to the first loading to rupture and subscript 2 refers to the second loading to rupture. This arrangement thus made possible two breaks per specimen.

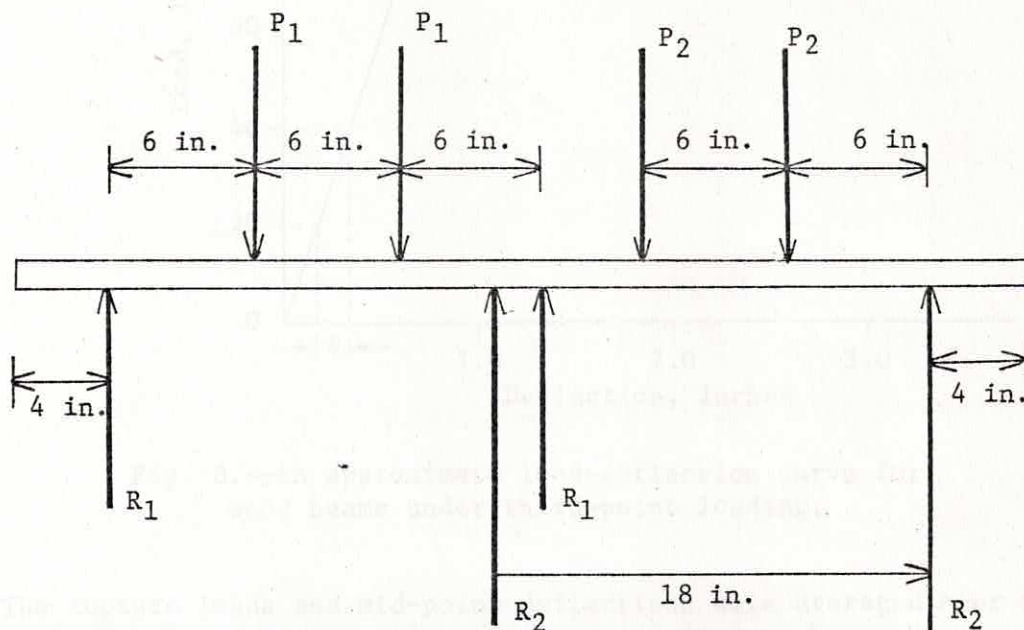


Fig. 7.--Specimen loading scheme.

The wood specimens were supported and loaded using the standard apparatus for third-point loading test for the wood beams described in (1). To determine the deflection at the mid-point of the span, the specimens were first loaded to a 20 lbs. initial load to remove any warping of the beam. A dial gage indicator was then placed under the mid-point of the beam span and the specimen loaded to 40 lbs. load to record the dial gage indicator reading

(Fig. 8). The specimens were then loaded to rupture and the maximum rupture load recorded. The second runs were made using the undamaged portion of the

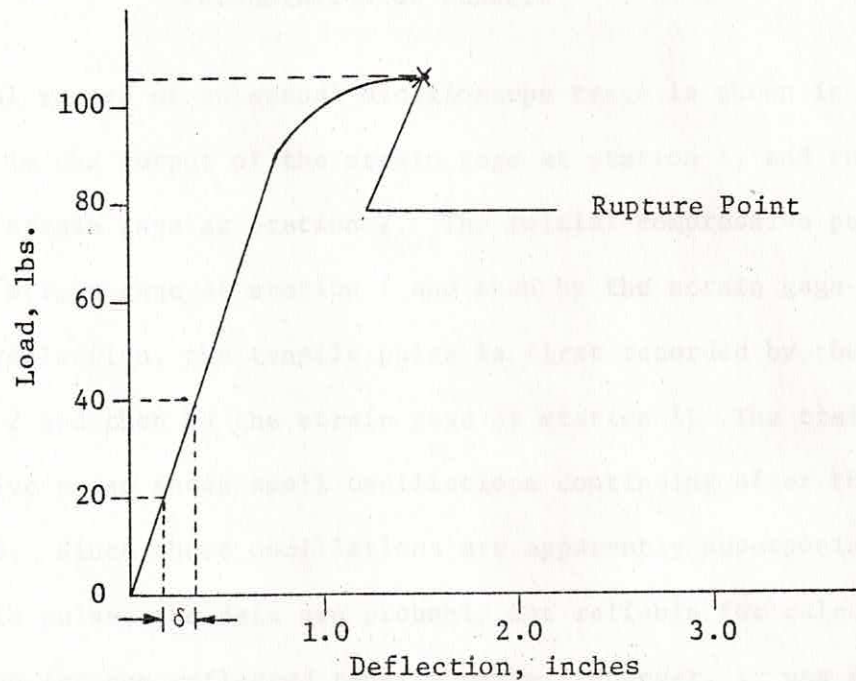


Fig. 8.--An approximate load-deflection curve for wood beams under third-point loading.

bar. The rupture loads and mid-point deflections were averaged over the two runs for data reduction. If the fracture of the bar occurred outside the constant moment section of the beam (Fig. 6), data for that run were rejected.

PRESENTATION OF RESULTS

A typical record of an actual oscilloscope trace is shown in Fig. 9. The upper trace is the output of the strain gage at station 1, and the bottom trace is of the strain gage at station 2. The initial compressive pulse is recorded by the strain gage at station 1 and then by the strain gage at station 2. After reflection, the tensile pulse is first recorded by the strain gage at station 2 and then by the strain gage at station 1. The trailing end of the compressive pulse shows small oscillations continuing after the strain pulse has passed. Since these oscillations are apparently superposing on the reflected tensile pulse, the data are probably not reliable for calculating internal friction for the reflected tensile pulse. However, it was still possible to calculate the pulse velocity in tension.

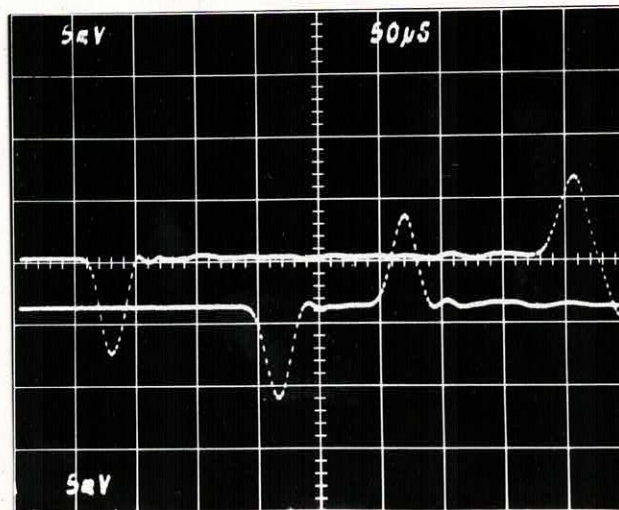


Fig. 9.--A typical oscilloscope record of a strain pulse.

All data from the dynamic response tests were averaged over two runs for each specimen, the second run being made with the bar ends turned over and the second end now bonded to the loading bar. Similarly, the modulus of rupture and static Young's modulus were computed from the average rupture load and average mid-point deflection of the beam over the two runs on each bar. A summary of measured and calculated quantities is given in Table 1 for each specimen.

Coefficients of correlation and regression lines' constants were computed for the modulus of rupture and various other measured quantities as well as between Young's modulus in compression and tension, and static and dynamic Young's modulus in compression. These values are listed in Table 2. All the regression lines were plotted and are shown in Fig. 10 and in Figs. 12 through 16. The average values of the modulus of rupture for various grain slopes are listed in Table 3.

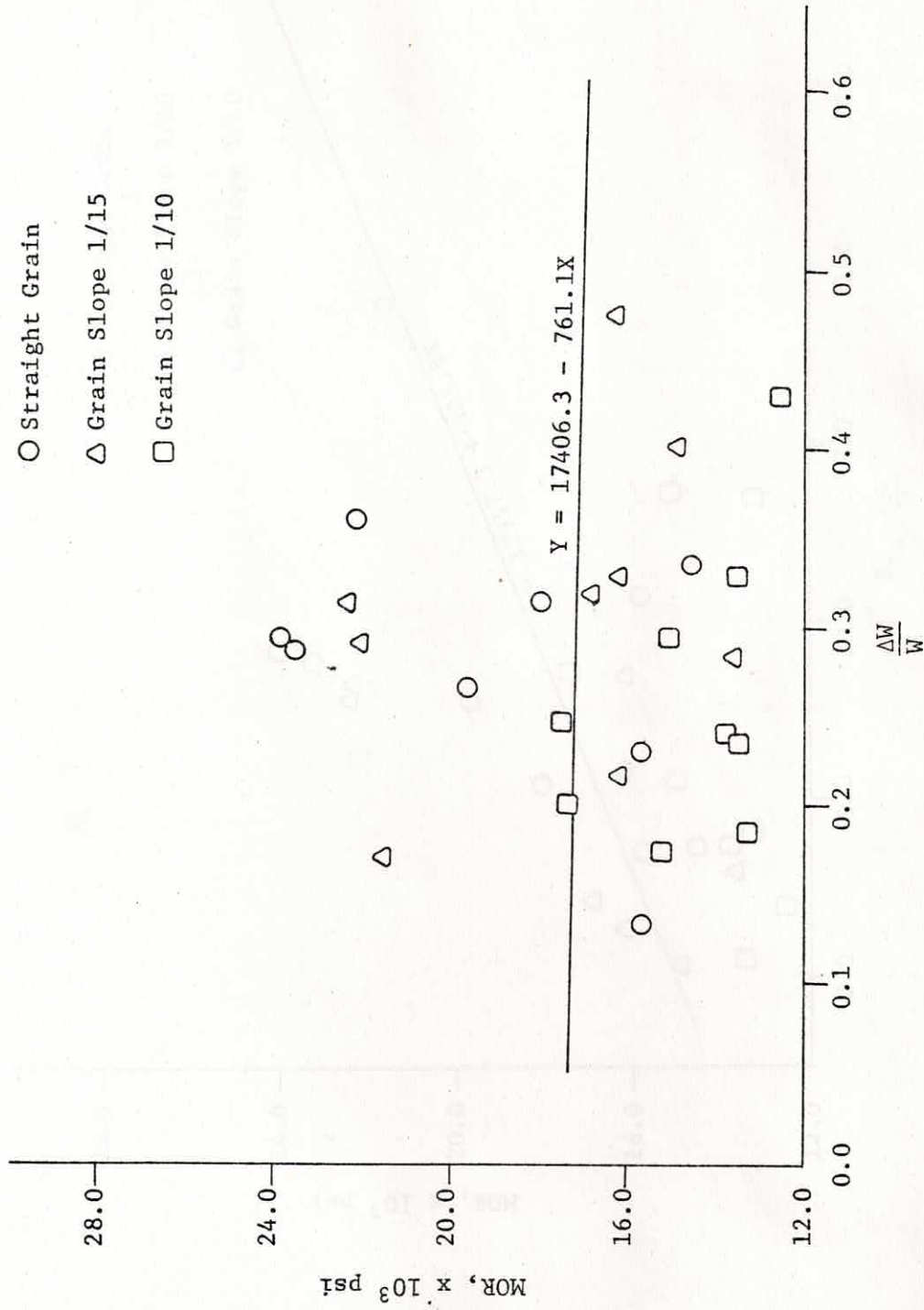


Fig. 10.--Regression of modulus of rupture and internal friction of wood.

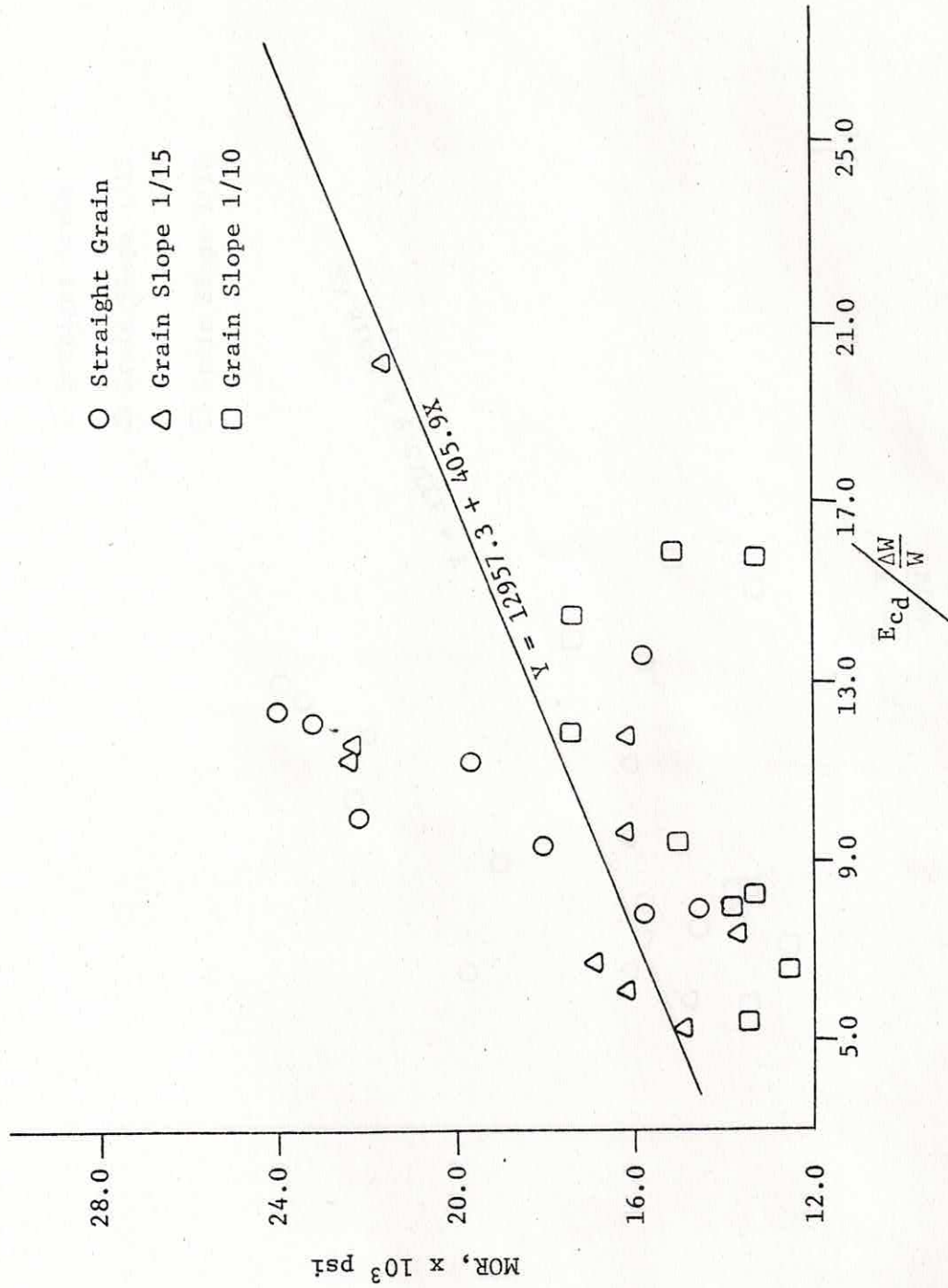


Fig. 11.--Regression of modulus of rupture and dynamic response parameters (I).

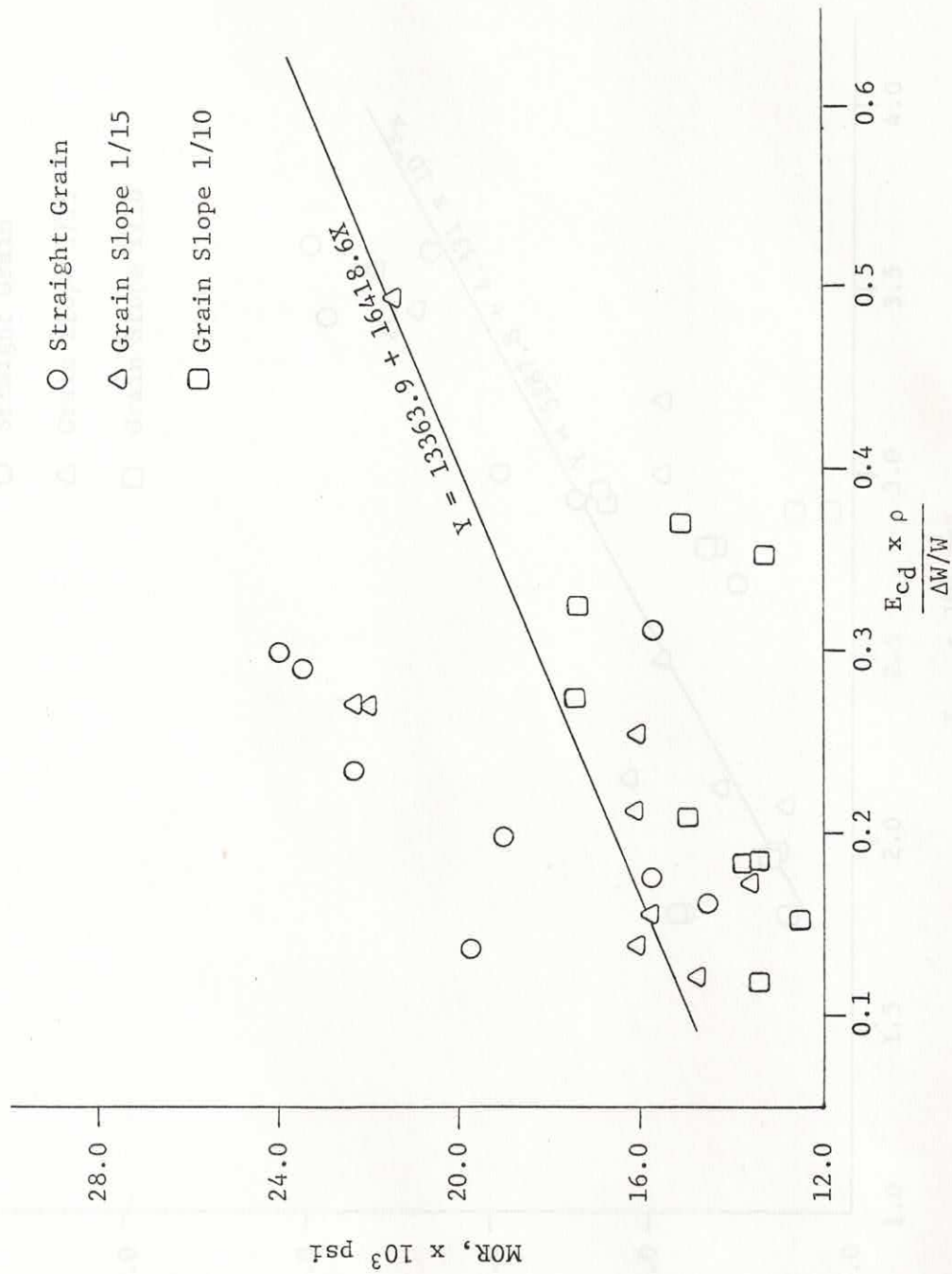


Fig. 12.--Regression of modulus of rupture and dynamic response parameters (II).

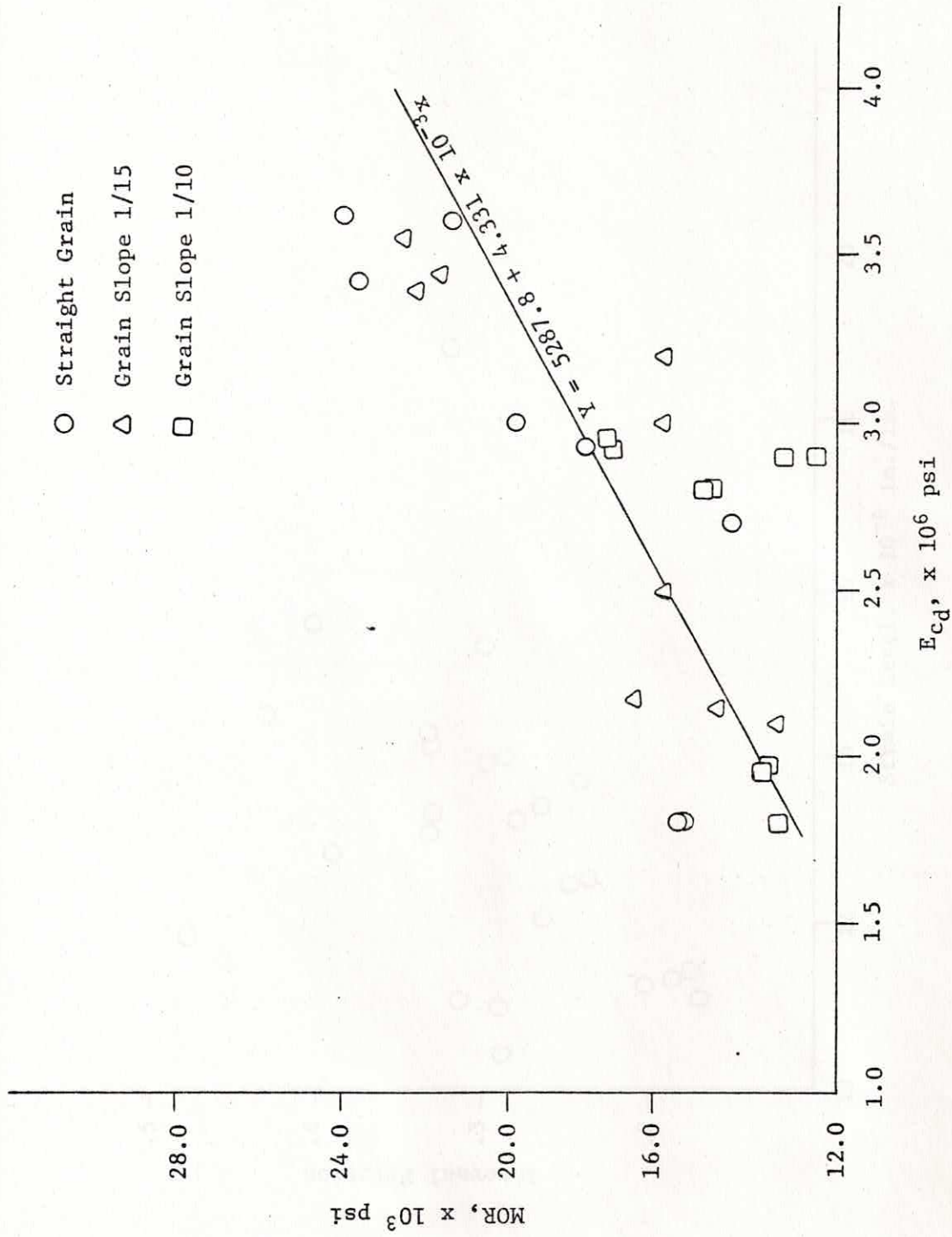


Fig. 13.--Regression of modulus of rupture and dynamic Young's modulus in compression.

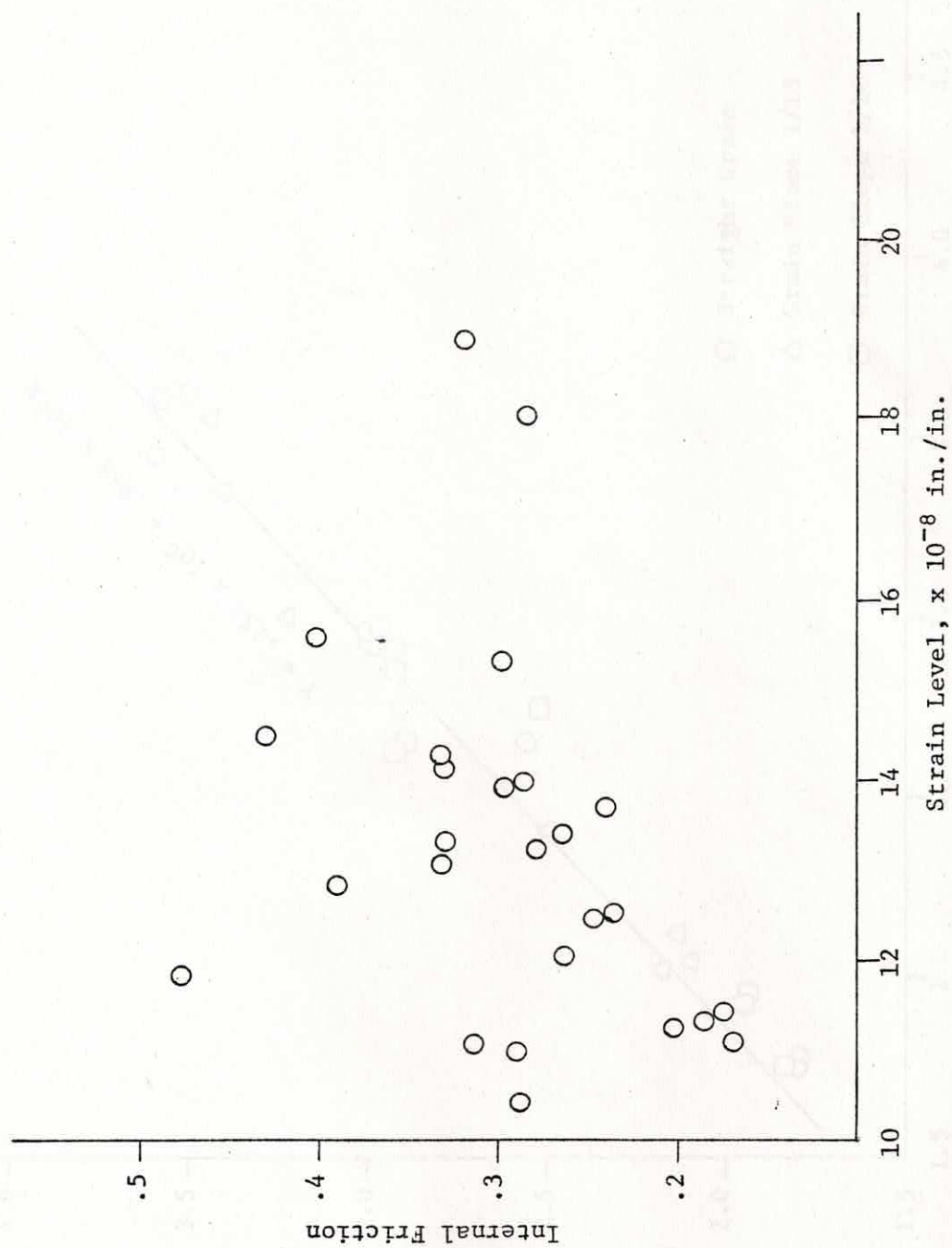


Fig. 14.--Plotted points for graph of maximum strain level against internal friction.

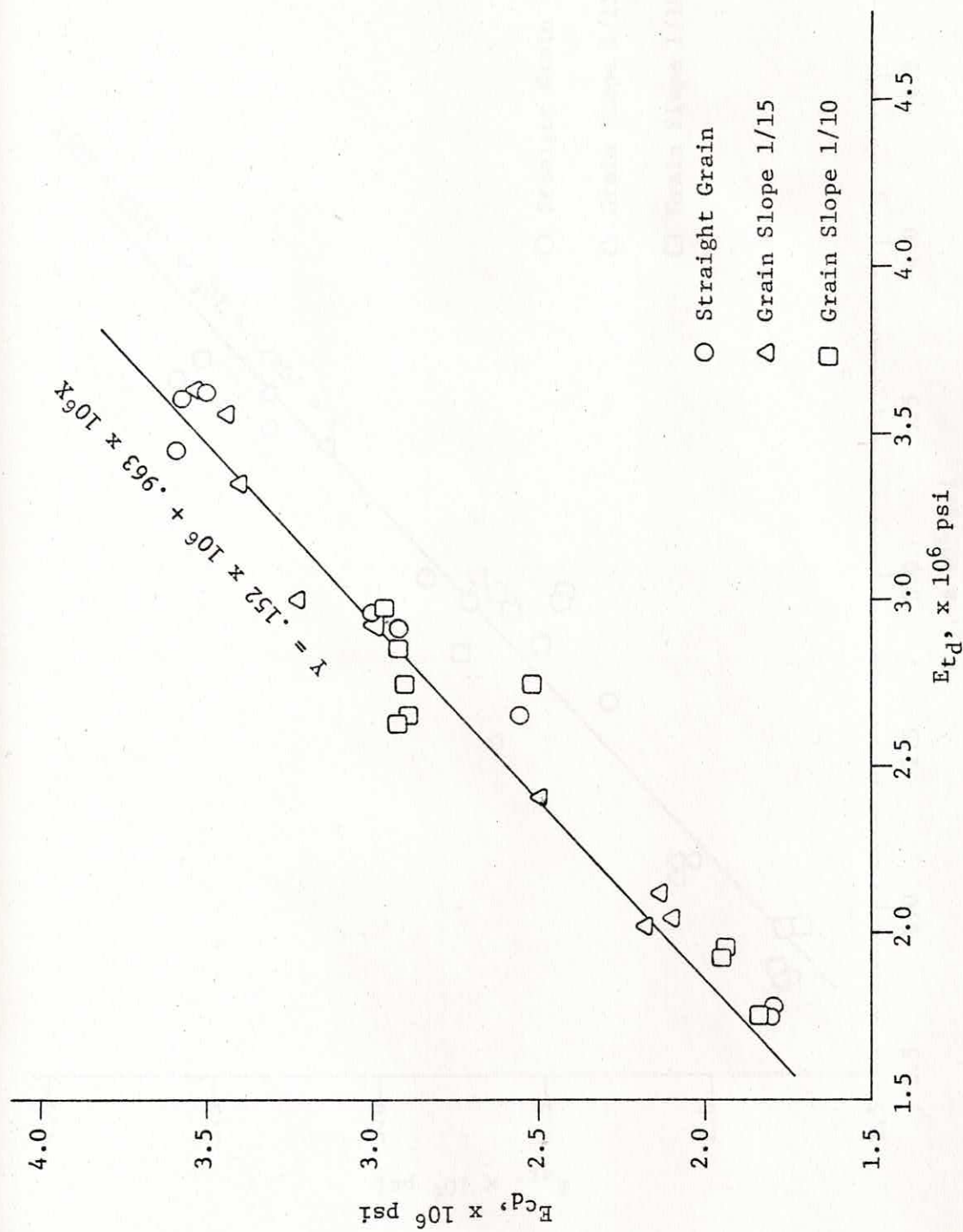


Fig. 15.--Regression of dynamic Young's modulus in compression and tension.

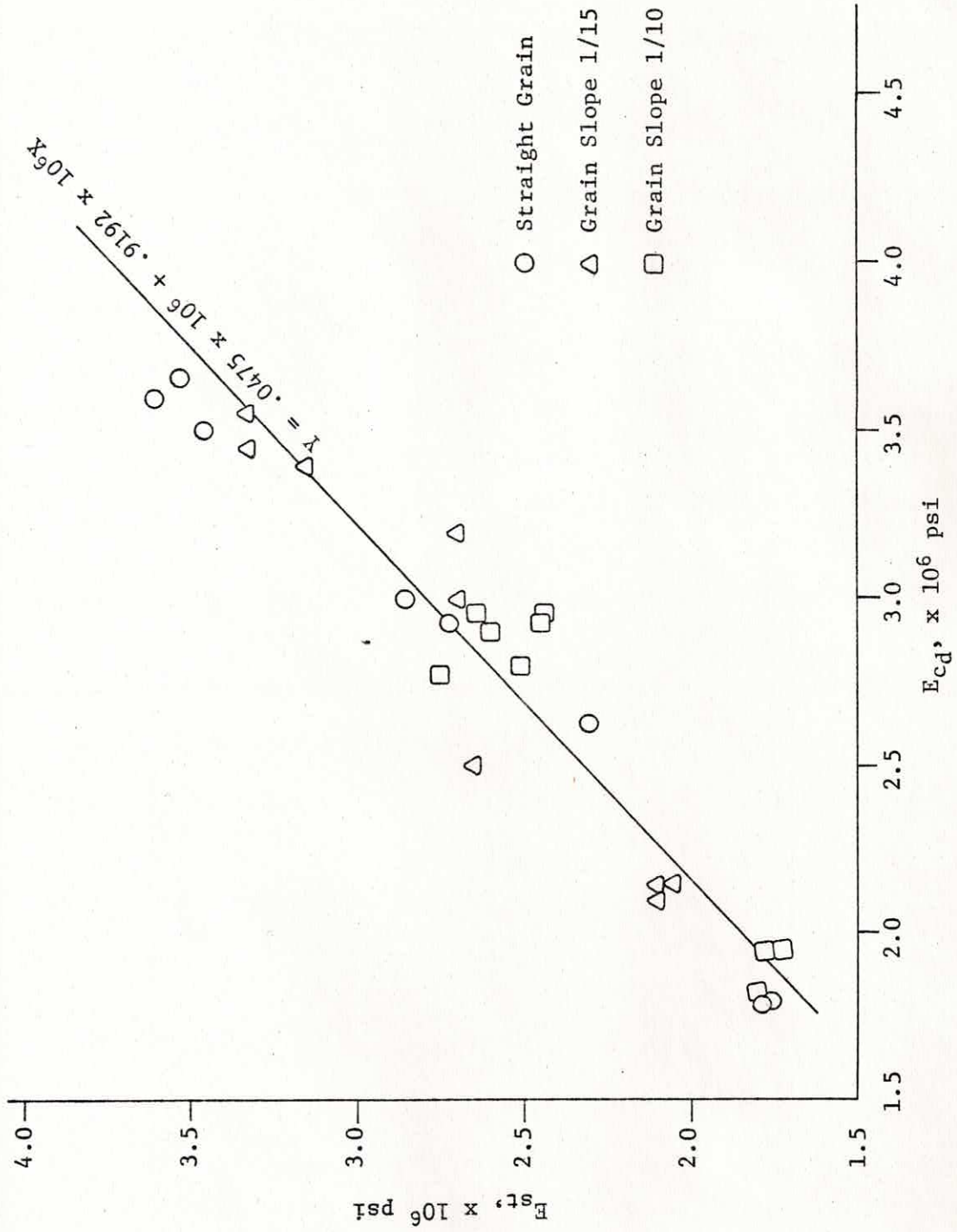


Fig. 16.--Regression of static and dynamic Young's modulus in compression.

TABLE 1.--Summary of calculated quantities for all runs

Specimen Bar No.	Grain Slope	Weight Density lb./in. ³ x 10 ⁻²	Maximum Strain Level Reached (μ s) ^a	Internal Friction $\left(\frac{\Delta W}{W}\right)$ Comp. ^a	Modulus of Rupture (MOR) x 10 ³ psi ^b
1A	0	2.15	1340	.265	19.80
1B	0	2.03	1307	.338	14.69
1C	0	2.11	1424	.313	18.14
2A	1/15	2.16	1326	.213	16.27
2B	1/15	2.21	1331	.331	16.27
2C	1/15	2.20	1183	.477	16.27
3A	1/10	2.23	1130	.184	13.32
3B	1/10	2.24	1124	.203	17.50
3C	1/10	2.28	1245	.247	17.53
4A	0	2.34	1203	.263	22.25
4B	0	2.42	1284	.392	23.98
4C	0	2.40	1038	.287	23.54
5A	1/15	2.31	1097	.293	22.10
5B	1/15	2.41	1107	.170	21.67
5C	1/15	2.38	1105	.314	22.46
6A	1/10	2.31	1140	.175	15.23
6B	1/10	2.40	1387	.296	15.05
6C	1/10	2.33	1447	.431	12.53
7A	0	2.26	1800	.281	15.84
7B	0	2.53	1529	.297	15.84
8A	1/15	2.30	1395	.282	13.75
8B	1/15	2.32	1893	.319	16.92
8C	1/15	2.29	1556	.403	14.98
9A	1/10	2.29	1371	.243	13.90
9B	1/10	2.23	1408	.329	13.54
9C	1/10	2.26	1255	.238	13.59

TABLE 1.--Continued

Dyn. Young's Modulus in Comp. (E_{cd}) $\times 10^6$ psi	Dyn. Young's Modulus in Tension (E_{td}) $\times 10^6$ psi	Static Young's Modulus (E_{st}) $\times 10^6$ psi	$\frac{E_{cd}-E_{st}}{E_{cd}} \times 100$ (%)	$\frac{E_{cd}-E_{st}}{E_{cd}} \times 100$ (%)
3.00	2.97	2.85	1.0	5.0
2.67	2.56	2.31	3.9	13.4
2.93	2.93	2.72	-0.2	6.9
2.51	2.41	2.68	3.9	-7.0
3.19	3.00	2.70	6.2	15.4
3.01	2.96	2.70	1.8	10.1
2.90	2.74	2.60	5.2	10.1
2.94	2.87	2.46	2.3	16.2
2.96	2.98	2.66	-0.7	10.2
3.60	3.45	3.60	4.0	0.1
3.66	3.61	3.53	1.2	3.5
3.49	3.62	3.46	-3.6	1.0
3.40	3.35	3.17	1.3	6.8
3.44	3.57	3.32	-3.7	3.3
3.55	3.64	3.33	-2.4	6.1
2.80	2.63	2.52	4.5	10.0
2.78	2.76	2.73	0.7	1.7
2.91	2.66	2.46	8.7	15.3
1.80	1.76	1.76	2.5	2.5
1.81	1.77	1.78	2.0	1.4
2.11	2.07	2.12	1.8	-0.2
2.17	2.12	2.10	2.2	3.0
2.16	2.05	2.07	5.0	3.9
1.94	1.92	1.78	1.0	8.4
1.79	1.76	1.78	1.8	0.3
1.97	1.94	1.73	1.4	12.0

^aCalculated quantities were averaged over the two runs made on each specimen.

^bQuantities calculated based on the average rupture load over two runs for each specimen.

TABLE 2.--Coefficients of correlation, R, and regression line constants, A and B, in regression equation $Y = A + Bx$ between various measured quantities

X	Y = MOR			Y = E_{td}			Y = E_{st}		
	R	A	B	R	A	B	R	A	B
$\Delta W/W$	-0.018	17.41×10^3	$-.76 \times 10^3$	--	--	--	--	--	--
$E_{cd} \frac{\Delta W}{W}$	0.420	12.96×10^3	0.41×10^3	--	--	--	--	--	--
$E_{cd} \times P \frac{\Delta W}{W}$	.418	13.36×10^3	16.42×10^3	--	--	--	--	--	--
E_{cd}	.757	5.29×10^3	4.33×10^{-3}	.990	$.152 \times 10^6$	$.963 \times 10^6$	.962	$.048 \times 10^6$	$.919 \times 10^6$

TABLE 3. Average values of modulus of rupture with various grain slopes

Grain Slope	Number of Specimens Tested	Averaged Modulus of Rupture (MOR)
0	8	19.2×10^3 psi
1/15	9	17.8×10^3 psi
1/10	9	14.6×10^3 psi

DISCUSSION OF RESULTS

The computed value of the coefficient of correlation between internal friction and modulus of rupture, -0.018 , showed no useful correlation between the two quantities. Because of the wide scatter of points about the regression line, Fig. 10, the regression line does not represent any defined trend or relationship between the internal friction and modulus of rupture. This value of the coefficient of correlation did not agree with those obtained by James (8) and Pellerin (17). James, working with longitudinal acoustic waves, obtained correlations between the damping capacity and the modulus of rupture, determined from the static bending tests, ranging from 0.24 to 0.71 . But in no case could the correlation be improved by combining the damping capacity with other wood parameters. He concluded that the damping capacity is of no value for predicting strength values of wood. James tested $1\frac{7}{8}$ by $1\frac{7}{8}$ by $39\frac{1}{2}$ -inch specimens which were either set in longitudinal vibrations by a light, sharp tap at their ends with a small hammer or by a continuous periodic force. The ratio of decay of the free vibrations, or the resonance amplitude for forced vibrations, was used as the criterion for calculating damping capacity. Pellerin tested wood boards at various moisture contents which were 2 inches thick with lengths ranging from 12 feet to 16 feet and widths from 6 inches to 12 inches in free transverse vibrations. Correlations obtained between the log decrement of the vibrations and the modulus of rupture, determined from static bending tests, ranged from 0.57 to 0.88 for various moisture content levels. These correlations were generally improved when log decrement was combined with dynamic Young's modulus ranging from 0.68 to 0.92 .

Trials were made in the present study to improve the correlation by combining internal friction with other parameters that influence modulus of rupture. A coefficient of correlation of 0.420 was obtained by combining internal friction with dynamic Young's modulus (Fig. 11). A correlation of 0.418 was obtained when internal friction was combined with dynamic Young's modulus and density of wood (Fig. 12). Also, correlation between dynamic Young's modulus and modulus of rupture was determined to be 0.757. This essentially implied that the internal friction had a negative contribution to the correlation of modulus of rupture with other properties of wood. These results agree with the results obtained by James (8). By far the best correlation was found to be with Young's modulus alone, Fig. 13.

In this study, the internal friction was determined from the attenuation of a propagating strain pulse that subjects the transverse cross sections of the bar to a uniformly distributed stress. However, the rupture modulus was determined from static bending test that subjects the transverse cross sections to a stress that varies linearly from tension to compression with zero stress at the neutral axis of the beam. This could be the main reason for which the results obtained in this study did not agree with that obtained by Pellerin.

The stress levels reached in this study were below the stress at the proportional limit for coast Douglas fir, which is approximately 8,000 psi. However, it was felt that unknown errors could have been introduced due to the comparatively high strain levels reached in this study than those used by James and Pellerin. But there is no conclusive proof that the strain level influences internal friction.

Figure 14 shows plotted points for strain level against internal friction. From this plot, it was not possible to establish a trend of any

kind of variation of internal friction with strain level. In this study the average strain levels reached were of the order of 1,300 μ s, with the projectile firing pressures kept between 4 to 5 psi. It was difficult, however, to control and keep the strain levels consistently constant throughout the testing of the specimens, since Young's modulus of the specimens varied over a wide range from specimen to specimen and even within the same specimen along its length. This could have introduced some unknown error. It is doubtful, however, that the correlation could have been improved significantly even if this problem was somehow circumvented.

The experiments conducted by Bennewitz and Rotger, as cited by (11), on German silver showed that internal friction is dependent upon the frequency of vibrations and that it reaches a maximum value at a particular frequency. However, for homogeneous non-metals, the mechanism by which elastic energy is converted into heat is different from that for metals. It is supposed that the specific loss is due to hysteresis phenomenon shown by the inelastic non-metals. In such a case, then, as has been postulated and proven for many materials by Gemant and Jackson, as cited by (11), at low frequencies, internal friction is independent of frequency. Later, Wegel and Walther proved, as cited by (11), that internal friction remains independent even at much higher frequencies also. The pulse frequencies generated in this study, however, were in the range of 10 to 11 KHz. It was considered that this frequency variation should not introduce any significant error in the data, since wood is a non-metallic material.

Table 3 shows the effect of grain slope on rupture modulus. Although grain slopes 0, 1/15, and 1/10 are not very high, the average modulus of rupture showed a marked decrease with increasing grain slope. This follows the

well-known phenomenon that wood strength parallel to the grain is approximately forty times greater than that normal to the grain.

Attempts were made to correlate various combinations of internal friction and other influencing parameters with modulus of rupture for sets of specimens with the same grain slopes. No marked improvement of any kind was observed. It was also observed that within the range of grain slopes studied, grain slope does not appear to have any influence on internal friction. Figure 15 shows plotted points for dynamic Young's modulus in tension against dynamic Young's modulus in compression. These quantities showed very good correlation of 0.99. Also it was determined that on the average, dynamic Young's modulus in compression is higher than that in tension by about 1.9%.

The correlation between dynamic Young's modulus and static Young's modulus was found to be 0.962, indicating a very strong relationship between them (Fig. 16). The average difference between the two was computed to be 6.3% with dynamic Young's modulus being higher than the static Young's modulus. This agreed well with the results reported by Pellerin (17).

In some of the specimen bars, it was observed that the amplitude of the strain pulse, instead of attenuating, accentuated after reflecting back from the free end as a tensile pulse. Table 4 lists dynamic response data for all such bars. The data listed are for the strain gage nearest to the free end and shows compressive and reflected tensile strain amplitudes. Run 2 was made with the bar ends reversed. This observed effect could not be completely understood. It was reasoned that since Young's modulus in compression is higher than in tension, it could cause such an effect. But this effect alone, however, did not explain it fully, since the increase in tensile strain was greater than what could be accounted for by this fact.

TABLE 4.--List of specimens showing higher amplitude of reflected tensile pulse than the initial compressive pulse

Bar No.	Run	Observed Compressive Strain ϵ_c , (μs)	Observed Tensile Strain ϵ_t , (μs)	$\frac{\epsilon_c - \epsilon_t}{\epsilon_c} \times 100$	$\frac{E_{cd} - E_{ct}}{E_{cd}} \times 100$
1B	1	985	1007	-2.2	3.9
	2	1124	1157	-2.9	
2B	1	1678	1708	-1.8	6.2
	2	1016	1163	-14.5	
2C	1	986	1050	-6.5	1.8
	2	795	855	-7.5	
5A	1	880	922	-4.8	1.3
	2	969	1008	-4.0	
5C	1	1585	1594	-0.6	2.4
	2	1017	1121	-10.2	
6C	1	1002	1068	-6.0	8.7
	2	1176	1211	-3.0	
7A	1	1642	1602	+2.5	2.5
	2	1307	1370	-4.8	

These tests should yield more consistent data because of the fact that the stress state in both loading situations would be the same.

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

Internal friction, as determined from the attenuation of a propagating one-dimensional stress pulse in wood bars, had no correlation with the rupture modulus of wood determined from the static bending tests. Hence, according to this study, internal friction cannot be used as a parameter to predict strength properties of wood. It appears from this work that within the range of grain slopes studied, grain slope does not influence internal friction.

There was a very good correlation, 0.962, between the dynamic Young's and static Young's modulus. The average value of dynamic Young's modulus was about 6.3% higher than that of static Young's modulus. Wood seems to be duplicating the behavior of isotropic materials in the sense that the dynamic Young's modulus in compression was higher than that in tension by about 1.9%.

It is suggested that future work to determine internal friction be done at very low strain levels. Also, it is suggested that if possible, the strain level be kept constant throughout the dynamic testing. This is advisable since no conclusive proof is available for the influence of strain level on internal friction of wood. Another interesting area of investigation would be to study the effect of strain level on internal friction itself. If a trend or relationship between strain level and internal friction is established, that might explain the poor correlation between internal friction and strength properties obtained in this study.

It is also suggested that the static tests, used to determine the strength properties of wood, be either uniaxial compression or tension tests.

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APPENDIX A

STRAIN-STRESS CONSTITUTIVE EQUATIONS FOR WOOD

In the tensorial form, generalized Hooke's Law is written as:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad [A1]$$

C_{ijkl} is a fourth-order tensor and represents 81 elements. It is known as the stiffness matrix. If it is assumed that the material is homogeneous so that these elements are constants and not functions of spatial coordinates, the number of independent constants reduces to 36. The above assumption implies that stress and strain tensors are symmetric, i.e.:

$$\sigma_{ij} = \sigma_{ji} \quad \text{and} \quad \epsilon_{ij} = \epsilon_{ji}$$

For convenience, Eq. [10] can now be written in the non-tensorial form:

$$\sigma_i = C_{ij} \epsilon_j \quad [A2]$$

where $i, j = 1, 2, 3, 4, 5, 6$; σ_i and ϵ_j being engineering rather than tensorial stresses and strains.

If the existence of a strain energy density function is assumed, it can be shown that $C_{ij} = C_{ji}$ (4) so that the number of independent constants reduces to 21.

Furthermore, if the material is orthotropic, the stiffness matrix has only nine independent constants in the stiffness matrix, as shown on the following page (4,12).

$$C_{ij} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{vmatrix} \quad [A3]$$

Figure A.1 shows a small piece of natural wood with yearly growth rings. Neglecting the heterogeneity and curvature of the yearly growth rings, three planes of construction symmetry can be recognized, which are also planes of

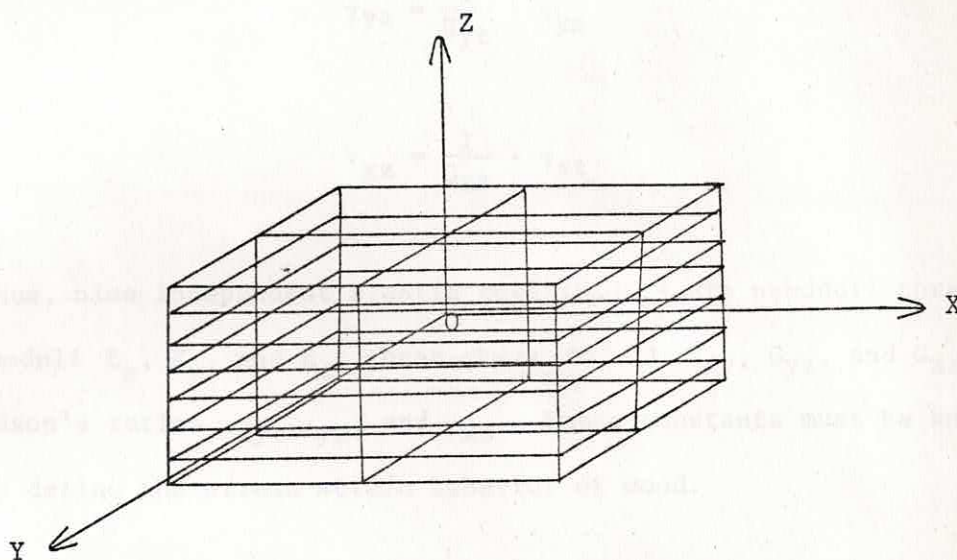


Fig. A.1.--Planes of elastic symmetry for an orthotropic material.

elastic symmetry. Plane YOZ, normal to the grain of wood, is called the longitudinal plane. Plane XOY is parallel to the grain and is called the tangential plane. Finally, plane XOZ, orthogonal to the first two planes, is called the radial plane. Referring to Fig. A.1, strain-stress constitutive equations can be written for wood as: (6,12)

$$\epsilon_x = \frac{1}{E_x} \sigma_x - \frac{\nu_{xy}}{E_y} \sigma_y - \frac{\nu_{xz}}{E_z} \sigma_z$$

$$\epsilon_y = -\frac{\nu_{xy}}{E_x} \sigma_x + \frac{1}{E_y} \sigma_y - \frac{\nu_{yz}}{E_z} \sigma_z$$

$$\epsilon_z = \frac{\nu_{xz}}{E_x} \sigma_x + \frac{\nu_{yz}}{E_y} \sigma_y + \frac{1}{E_z} \sigma_z$$

$$\gamma_{xy} = \frac{1}{G_{xy}} \cdot \tau_{xy}$$

$$\gamma_{yz} = \frac{1}{G_{yz}} \cdot \tau_{yz}$$

$$\gamma_{xz} = \frac{1}{G_{xz}} \cdot \tau_{xz}$$

Thus, nine independent elastic coefficients are needed: three Young's moduli E_x , E_y , and E_z , three shear moduli G_{xy} , G_{yz} , and G_{xz} , and three Poisson's ratios ν_{xy} , ν_{yz} , and ν_{xz} . These constants must be known to completely define the stress-strain behavior of wood.

APPENDIX B

ERROR CORRECTION IN THE DATA

Recall Eq. [4], which is used to evaluate attenuation constant:

$$\alpha = \frac{1}{x} \ln \frac{\sigma_0}{\sigma}$$

Assuming that the Young's modulus, E , of wood is constant throughout the specimen length, the ratio:

$$\frac{\sigma_0}{\sigma} = \frac{\epsilon_0}{\epsilon}$$

where ϵ_0 and ϵ are the corresponding strains for the stresses σ_0 and σ .

Now consider a case in which local values of Young's modulus, E , at strain gage station 1 is different than at strain gage station 2 (Fig. B.1).

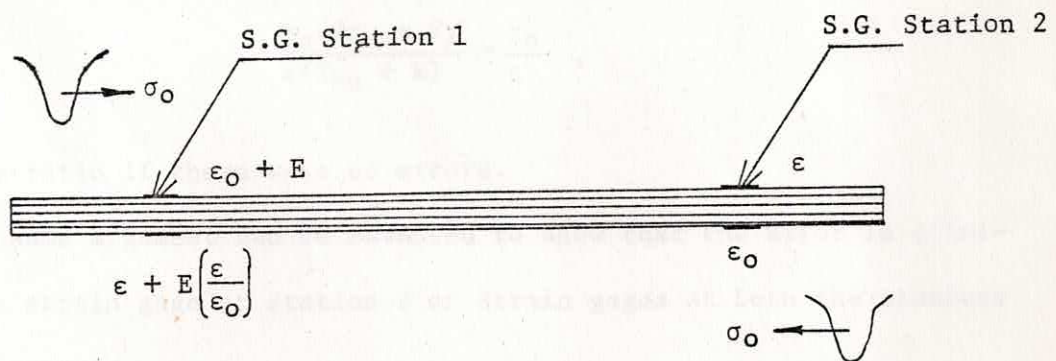


Fig. B.1.--Strain gage output error for a strain pulse traveling from station 1 to station 2 and from station 2 and station 1.

Then the ratio (ϵ_0/ϵ) required to evaluate the attenuation constant will not be true, since the strain amplitudes are being affected by the difference in E at the two stations.

Next, let E be an error, proportional to the peak strain amplitude, which is being introduced in the output of strain gage at station 1. This value of the error could be due to difference in strain gage factor, possibly poor strain gage bonding on the wood surface as well as due to the difference in Young's modulus at the two stations. Also suppose that the strain gage at station 2 recorded the actual strain ϵ , after attenuation for a strain pulse propagating from station 1 to station 2. Therefore, for such a strain pulse the signal from the strain gages will be $(\epsilon_0 + E)$ and (ϵ) . The output from the strain gages for a strain pulse of the same initial amplitude and traveling from station 2 to station 1 will be (ϵ_0) and $\left| \epsilon + E \left(\frac{\epsilon}{\epsilon_0} \right) \right|$

The ratio of the average of the two runs is then:

$$\frac{\epsilon_0 + (\epsilon_0 + E)}{\epsilon + \left| \epsilon + \epsilon \left(\frac{E}{\epsilon_0} \right) \right|}$$

or

$$\frac{\epsilon_0(2\epsilon_0 + E)}{\epsilon(2\epsilon_0 + E)} = \frac{\epsilon_0}{\epsilon} ,$$

which is the ratio if there were no errors.

The same argument can be advanced to show that the error is eliminated if the strain gage at station 2 or strain gages at both the stations have faulty outputs.