

Sci
WSU
C6
1966
I 8

IYENGAR

DEFLECTION CURVES FOR
COLUMNS BENDING ABOUT
THE WEAK AXIS

DEFLECTION CURVES FOR COLUMNS

BENDING ABOUT THE WEAK AXIS

By

SAMPATH N.S. IYENGAR



A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN

CIVIL ENGINEERING

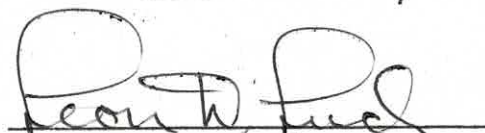
WASHINGTON STATE UNIVERSITY

1966

To the Resident Instructional Staff:

The members of the Committee appointed to
examine the thesis of SAMPATH N.S. IYENGAR find it satis-
factory and recommend that it be accepted.


Chairman





ACKNOWLEDGMENTS

The constant and encouraging guidance of the thesis committee consisting of Professors Kenneth A. Bush, Leon D. Luck, and Loren B. Almy, Chairman is gratefully acknowledged. My thanks are also due to Mrs. Nancy Westbrook for the patient secretarial assistance.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS.	iii
LIST OF TABLES	v
LIST OF ILLUSTRATIONS	vi
INTRODUCTION	1
SCOPE OF THE PRESENT THESIS.	1
COLUMN DEFLECTION CURVES	2
ASSUMPTIONS.	2
PROCEDURE TO OBTAIN THE CDC'S.	3
Illustration of the procedure (8WF31 section).	5
$P = 0.2 P_y$	7
$P = 0.6 P_y$	8
GRAPHICAL REPRESENTATION OF THE CDC'S.	9
STRENGTHS OF COLUMNS	9
SUMMARY.	15
LITERATURE CITED	17
APPENDIX	
A. NOMENCLATURE FOR TEXT.	18
B. NOMENCLATURE FOR COMPUTER ANALYSIS	21
C. COMPUTER ANALYSIS.	23
ABSTRACT	40

LIST OF TABLES

Table	Page
1. Comparative Values.	15
2. Program for $P = 0.2P_y$ and $\alpha_0 = 0.01, 0.02, \dots$	24
3. Results for $P = 0.2P_y$ and $\alpha_0 = 0.01, 0.02, \dots$	26
4. Program for $P = 0.2P_y$ and $\alpha_0 = 0.075. \dots$	30
5. Results for $P = 0.2P_y$ and $\alpha_0 = 0.075. \dots$	32
6. Program for $P = 0.6P_y$ and $\alpha_0 = 0.01, 0.02, \dots$	33
7. Results for $P = 0.6P_y$ and $\alpha_0 = 0.01, 0.02, \dots$	35
8. Program for $P = 0.6P_y$ and $\alpha_0 = 0.065. \dots$	37
9. Results for $P = 0.6P_y$ and $\alpha_0 = 0.065. \dots$	39

LIST OF ILLUSTRATIONS

Figure		Page
1.	Column Deflection Curves	2
2.	Residual Stress Pattern.	3
3.	Segment on CDC	4
4.	8WF31 Section.	5
5.	Column Deflection Curve.	9
6.	M/M_{pc} vs. α and Z/r vs. α Curves	10
7.	M/M_{pc} vs. α and Z/r vs. α Curves	11
8.	M/M_{pc} vs. θ Curve for $P = 0.2P_y$	13
9.	M/M_{pc} vs. θ Curve for $P = 0.6P_y$	14

INTRODUCTION

The moment-carrying capacity of steel sections is reduced from M_p to M_{pc} in the presence of axial thrust over the section. A further reduction occurs in a column because of the buckling effect, the amount of reduction depending upon the slenderness ratio of the column.

Strengths of columns bending about the strong axis have been ascertained theoretically and these values compared with test results (1, 2, 3). In general, the individual bents of a framed building resisting moments due to either wind or earthquake forces or unbalanced loads are to be investigated for such strength. Cases may arise, however, where moment capacities of columns bending about the weak axis due to any one or any combination of the above causes govern the sizes of columns. The present investigation seeks to provide a semigraphical solution to the problem, using the concept of Column Deflection Curves, and compares the results with similar curves for bending about the strong axis.

SCOPE OF THE PRESENT THESIS

Column Deflection Curves based on computed values have been obtained for columns bending about the weak axis under the influence of increasing end-moments (symmetrical single curvature) and in the presence of constant axial thrust (1) $P = 0.2 P_y$, (2) $P = 0.6 P_y$. Strengths of columns bending about the weak axis and with slenderness ratios 20, 30, and 40 are estimated from these curves.

COLUMN DEFLECTION CURVES

A column with any ratio of end moments may be considered as a part of the Column Deflection Curve or CDC (1). CDC's may be developed for various assumed values of α_0 , the end rotation corresponding to a particular assumed value of P , the axial thrust.

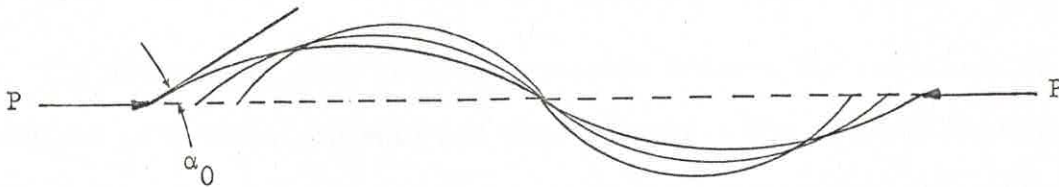


Fig. 1--Column Deflection Curves

The length of the CDC decreases for increasing values of α_0 . The limiting value of α_0 is reached when the moment at the center of the CDC (quarter wave length from end) equals M_{pc} .

ASSUMPTIONS

Rolled WF shapes are subject to residual stresses over the cross section as a result of the cooling of the material after hot rolling in the mills. The magnitude, and to a minor degree the nature of these stresses, differ with each cross section. In general, however, the following pattern of stresses may be assumed to exist (1).

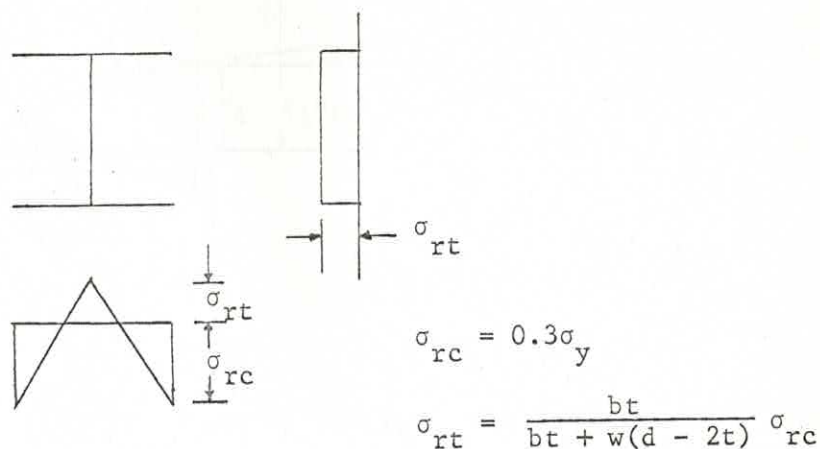


Fig. 2--Residual Stress Pattern

The presence of such residual stresses affects the curvature of a section subject to external moments and axial thrust. The effects are more pronounced for bending about the weak axis than for bending about the strong axis (3). Results are available for the 8WF31 section and it has also been shown that this section is quite typical of most steel rolled sections. For the present work, therefore, M-P- ϕ curves as derived (3) have been assumed to hold for bending about the weak axis.

Further assumptions are: (1) Plane sections remain plane. (2) Material exhibits ideal elastic-plastic σ - ϵ curve (no strain hardening). (3) Flexure is in one plane only. (4) Axial force is applied first, then moment. (5) Deflections are small. (6) Members are prismatic. (7) No strain reversal occurs. (8) No initial crookedness exists.

PROCEDURE TO OBTAIN THE CDC'S

The axial thrust P and the end rotation α_0 are initially assumed. Subdivisions ρ_j are selected along the thrust line and the deflected shape within each segment is considered to be a circular arc.

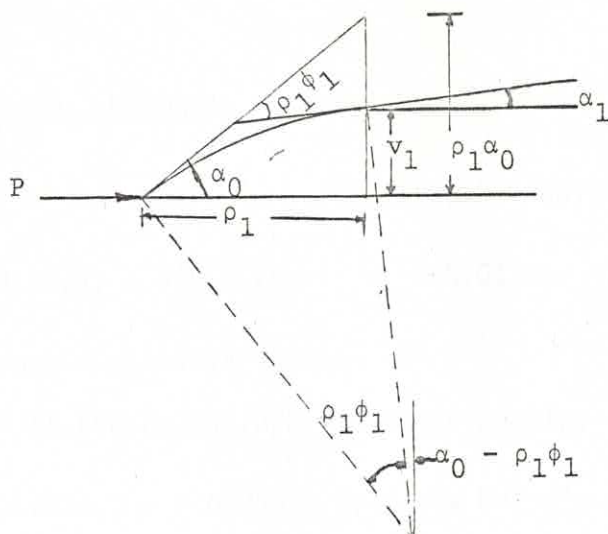


Fig. 3--Segment on CDC

Neglecting higher-order terms,

$v_1 = \rho_1 \alpha_0 - \rho_1^2 \phi_1 / 2$ and $\alpha_1 = \alpha_0 - \rho_1 \phi_1$ at the end of the first segment.

$v_2 = v_1 + \alpha_1 \rho_2 - \rho_2^2 \phi_2 / 2$ and $\alpha_2 = \alpha_1 - \rho_2 \phi_2$ at the end of the second segment.

$v_j = v_{j-1} + \alpha_{j-1} \rho_j - \rho_j^2 \phi_j / 2$ and $\alpha_j = \alpha_{j-1} - \rho_j \phi_j$ at the end of the j th segment.

$\phi_1, \phi_2, \dots, \phi_j$ are the curvatures of the segments corresponding to the mean moments in the segments.

The mean moment in segment 1 is $M_{m1} = P \alpha_0 \rho_1 / 2$;

The mean moment in segment 2 is $M_{m2} = P v_1 + P \alpha_1 \rho_2 / 2$;

The mean moment in segment j is $M_{mj} = P v_{j-1} + P \alpha_{j-1} \rho_j / 2$.

The process of numerical integration is continued till $\alpha_n = 0$.

$\rho_1, \rho_2, \dots, \rho_j$ are chosen as equal subdivisions for convenience. Values of ρ_j between r and $4r$ ($r < \rho_j < 4r$) are known to give reasonably accurate results without excessive computations (4). In the neighborhood of $\alpha = 0$, however, ρ is reduced suitably depending on the degree of accuracy desired.

The length of the quarter wave-length is $\sum_{j=1}^n \rho_j$ and represents half the

length of the CDC.

Illustration of the procedure
(8WF31 section):

$$P = 0.2 P_y = 0.2(A)\sigma_y = 0.2(9.12)36.0 = 65.664 \text{ kips. } \alpha_0 = 0.01 \text{ radian.}$$

$$M_y = S_y \sigma_y = 9.2(36.0) = 331.2 \text{ kips. } r_y = 2.01 \text{ in. Therefore, } 4r_y = 8.04 \text{ in.}$$

$\rho = 8 \text{ in.}$ is hence a suitable value.

$$M_{m1} = P\alpha_0\rho_1/2 = 65.664(0.01)8.0/2 = 2.62656 \text{ in-kips.}$$

$$M_{m1}/M_y = 2.62656/331.2 = 0.00793. \text{ From the M-P-}\phi \text{ curve (3), for } P = 0.2 P_y$$

and $M_{m1}/M_y = 0.00793$, the value of $\phi_1/\phi_y = 0.00793$. $E\phi_y = 2\sigma_y/d$. Therefore,

$$\phi_y = 2\sigma_y/Ed. \quad \phi_y = 2(36.0)/30,000(8) = 0.0003 \text{ radian.}$$

$$\text{Therefore, } \phi_1 = 0.00793(0.0003) = 0.000002379 \text{ radian.}$$

$$\alpha_1 = \alpha_0 - \rho_1\phi_1 = 0.01 - 8.0(0.000002379) = 0.00998 \text{ radian.}$$

$$v_1 = \rho_1\alpha_0 - \rho_1^2\phi_1/2 = 8(0.01) - 64(0.000002379)/2 = 0.07992 \text{ in.}$$

$$M_1 = \text{Moment at the end of the first segment} = Pv_1 = 65.664(0.07992) = 5.248 \text{ in-k.}$$

The 8WF31 section bending about the weak axis is considered to consist of two rectangles, the web being neglected.

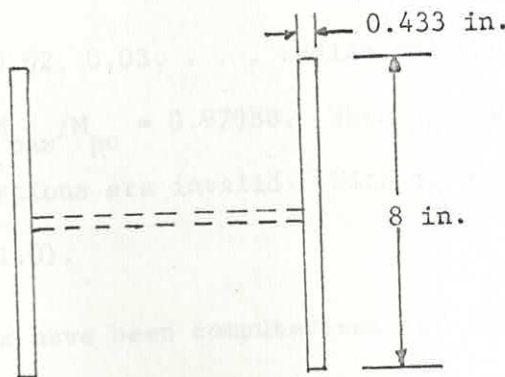


Fig. 4--8WF31 Section

Therefore, $M_{pc} = M_p [1 - (P/P_y)^2] = M_p (1 - 0.04) = 0.96 M_p$.

$$M_p = 2(36.0)0.433(64/4) = 576.0(0.866) = 498.816 \text{ in-k.}$$

$$M_1/M_{pc} = 5.248/(0.96)498.816 = 0.01096. \quad \sum_{j=1}^1 \rho_j = 8 \text{ in.}$$

$$M_{m2} = Pv_1 + P\alpha_1\rho_2/2 = (0.2)9.12(36.0)0.07992 + 0.2(9.12)36.0(0.00998)8.0/2 \\ = 7.8693 \text{ in-k.} \quad M_{m2}/M_y = 0.02376. \quad \text{Therefore, } \phi_2/\phi_y = 0.02376.$$

$$\phi_2 = 0.02376(0.0003) \text{ radian} = 0.000007128 \text{ radian.}$$

$$\alpha_2 = \alpha_1 - \rho_2\phi_2 = 0.00998 - 8(0.000007128) = 0.00992 \text{ radian.}$$

$$v_2 = v_1 + \alpha_1\rho_2 - \rho_2^2\phi_2/2 = 0.07992 + 0.00998(8) - 64(0.000007128)/2 = 0.1595 \text{ in.}$$

$$M_2 = Pv_2 = 0.2(36)(9.12)0.1595 = 10.506 \text{ in-kips.}$$

$$M_2/M_{pc} = 0.02188. \quad \sum_{j=1}^2 \rho_j = 8 + 8 = 16 \text{ in.}$$

Continuing in this manner,

$$\alpha_{25} = 0.00028, v_{25} = 1.29552 \text{ in.}, M_{25}/M_{pc} = 0.17765, \sum_{j=1}^{25} \rho_j = 200 \text{ in.}$$

If $\rho_{26} = 8 \text{ in.}$, α_{26} turns out to be negative. ρ_{26} is hence chosen as 1 in. Repeating the process, $\alpha_{28} = 0.00005$, while $\alpha_{29} = -0.00003$.

$$\sum_{j=1}^{28} \rho_j = 203 \text{ in.} \quad \sum_{j=1}^{29} \rho_j = 204 \text{ in.} \quad \text{The half length of the CDC may hence be taken}$$

as 203.5 in., approximately.

Next, α_0 is assumed as 0.02, 0.03, . . . radian and the process repeated.

When $\alpha_0 = 0.07$ radian, $M_{\max}/M_{pc} = 0.97988$. When $\alpha_0 = 0.08$ radian the ratio exceeds 1.00 and the calculations are invalid. With $\alpha_0 = 0.075$ radian,

$$M_{\max}/M_{pc} = 0.99899 \text{ (almost 1.0).}$$

All the calculations have been computerised for the two cases

(1) $P = 0.2 P_y$ and (2) $P = 0.6 P_y$, and end rotations in increments of 0.01 radian beginning with $\alpha_0 = 0.01$ radian.

It was felt necessary to use an increment of $\alpha_0 = 0.005$ radian at the final stage for $P = 0.2 P_y$. The maximum value of α_0 for the case $P = 0.2 P_y$ is, hence, $\alpha_0 = 0.075$ radian.

For the case $P = 0.6 P_y$, however, with $\alpha_0 = 0.06$ radian, the ratio of moments is 0.98086 and with $\alpha_0 = 0.065$ the ratio exceeds 1.0. Hence, the maximum value of α_0 is taken as 0.06 radian.

The M-P- ϕ curves (3) have been approximated by the following sets of straight lines for convenience in computer analysis:

$$\underline{P = 0.2 P_y}$$

$\phi/\phi_y = M/M_y$	if	$0.000 < M/M_y < 0.653$
$\phi/\phi_y = 0.653 + (M/M_y - 0.653)0.606/0.347$	if	$0.653 < M/M_y < 1.000$
$\phi/\phi_y = 1.259 + (M/M_y - 1.000)0.446/0.150$	if	$1.000 < M/M_y < 1.150$
$\phi/\phi_y = 1.705 + (M/M_y - 1.150)0.295/0.077$	if	$1.150 < M/M_y < 1.227$
$\phi/\phi_y = 2.000 + (M/M_y - 1.227)0.402/0.059$	if	$1.227 < M/M_y < 1.286$
$\phi/\phi_y = 2.402 + (M/M_y - 1.286)0.598/0.032$	if	$1.286 < M/M_y < 1.318$
$\phi/\phi_y = 3.000 + (M/M_y - 1.318)1.000/0.037$	if	$1.318 < M/M_y < 1.355$
$\phi/\phi_y = 4.000 + (M/M_y - 1.355)2.000/0.041$	if	$1.355 < M/M_y < 1.396$
$\phi/\phi_y = 6.000 + (M/M_y - 1.396)3.000/0.045$	if	$1.396 < M/M_y < 1.441$
$\phi/\phi_y = 9.000 + (M/M_y - 1.441)4.000/0.049$	if	$1.441 < M/M_y$

$$P = 0.6 P_{y-}$$

$\frac{\phi}{\phi_y} = \frac{M}{M_y}$	if	$\frac{0.00}{55.00} < \frac{M}{M_y} < \frac{12.75}{55.00}$
$\frac{\phi}{\phi_y} = \frac{2.60}{11.20} + \left(\frac{M}{M_y} - \frac{12.75}{55.00}\right) \frac{3.65/11.20}{9.25/55.00}$	if	$\frac{12.75}{55.00} < \frac{M}{M_y} < \frac{22.00}{55.00}$
$\frac{\phi}{\phi_y} = \frac{6.25}{11.20} + \left(\frac{M}{M_y} - \frac{22.00}{55.00}\right) \frac{4.95/11.20}{7.75/55.00}$	if	$\frac{22.00}{55.00} < \frac{M}{M_y} < \frac{29.75}{55.00}$
$\frac{\phi}{\phi_y} = \frac{11.20}{11.20} + \left(\frac{M}{M_y} - \frac{29.75}{55.00}\right) \frac{5.00/11.20}{4.75/55.00}$	if	$\frac{29.75}{55.00} < \frac{M}{M_y} < \frac{34.50}{55.00}$
$\frac{\phi}{\phi_y} = \frac{16.20}{11.20} + \left(\frac{M}{M_y} - \frac{34.50}{55.00}\right) \frac{6.20/11.20}{4.40/55.00}$	if	$\frac{34.50}{55.00} < \frac{M}{M_y} < \frac{38.90}{55.00}$
$\frac{\phi}{\phi_y} = \frac{22.40}{11.20} + \left(\frac{M}{M_y} - \frac{38.90}{55.00}\right) \frac{5.75/11.20}{2.35/55.00}$	if	$\frac{38.90}{55.00} < \frac{M}{M_y} < \frac{41.25}{55.00}$
$\frac{\phi}{\phi_y} = \frac{28.15}{11.20} + \left(\frac{M}{M_y} - \frac{41.25}{55.00}\right) \frac{5.45/11.20}{1.75/55.00}$	if	$\frac{41.25}{55.00} < \frac{M}{M_y} < \frac{43.00}{55.00}$
$\frac{\phi}{\phi_y} = \frac{33.60}{11.20} + \left(\frac{M}{M_y} - \frac{43.00}{55.00}\right) \frac{11.2/11.20}{2.00/55.00}$	if	$\frac{43.00}{55.00} < \frac{M}{M_y} < \frac{45.00}{55.00}$
$\frac{\phi}{\phi_y} = \frac{44.80}{11.20} + \left(\frac{M}{M_y} - \frac{45.00}{55.00}\right) \frac{11.2/11.20}{1.00/55.00}$	if	$\frac{45.00}{55.00} < \frac{M}{M_y} < \frac{46.00}{55.00}$
$\frac{\phi}{\phi_y} = \frac{56.00}{11.20} + \left(\frac{M}{M_y} - \frac{46.00}{55.00}\right) \frac{11.2/11.20}{0.50/55.00}$	if	$\frac{46.00}{55.00} < \frac{M}{M_y} < \frac{46.50}{55.00}$
$\frac{\phi}{\phi_y} = \frac{67.20}{11.20} + \left(\frac{M}{M_y} - \frac{46.50}{55.00}\right) \frac{11.2/11.20}{0.25/55.00}$	if	$\frac{46.50}{55.00} < \frac{M}{M_y} < \frac{46.75}{55.00}$
$\frac{\phi}{\phi_y} = \frac{78.40}{11.20} + \left(\frac{M}{M_y} - \frac{46.75}{55.00}\right) \frac{11.2/11.20}{0.13/55.00}$	if	$\frac{46.75}{55.00} < \frac{M}{M_y} < \frac{\infty}{55.00}$

GRAPHICAL REPRESENTATION OF THE CDC'S

The results of computer analysis have been reduced to graphical form by drawing the M/M_{pc} vs. α and Z/r vs. α curves. (Figs. 6,7)

Z is the distance measured from the center of a half-wave length as indicated in Fig. 5. The purpose of nondimensionalising M and Z is to make the end results applicable to all WF shapes and for various values of σ_y .

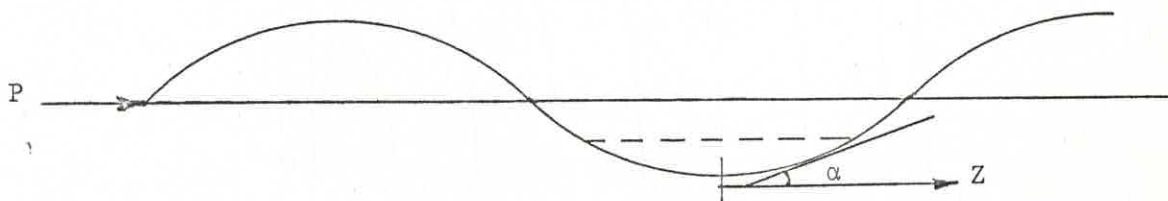


Fig. 5.--Column Deflection Curve

STRENGTHS OF COLUMNS

As mentioned before, a column with any ratio of end moments may be considered as a suitably chosen part of the CDC. A column bent in symmetrical single curvature (and hence equal but opposite end-moments) may be thought of as the portion of the CDC shown dotted in Fig. 5, since the zero slope at the middle of the column may be matched with the zero slope at the middle of the half-wave length of the CDC. Hence, the length L of the given column corresponds to $2Z$ on the CDC.

Knowing L , and hence Z , a horizontal line is drawn through the Z/r -value to cut the Z/r vs. α curves at several points. These points of inter-

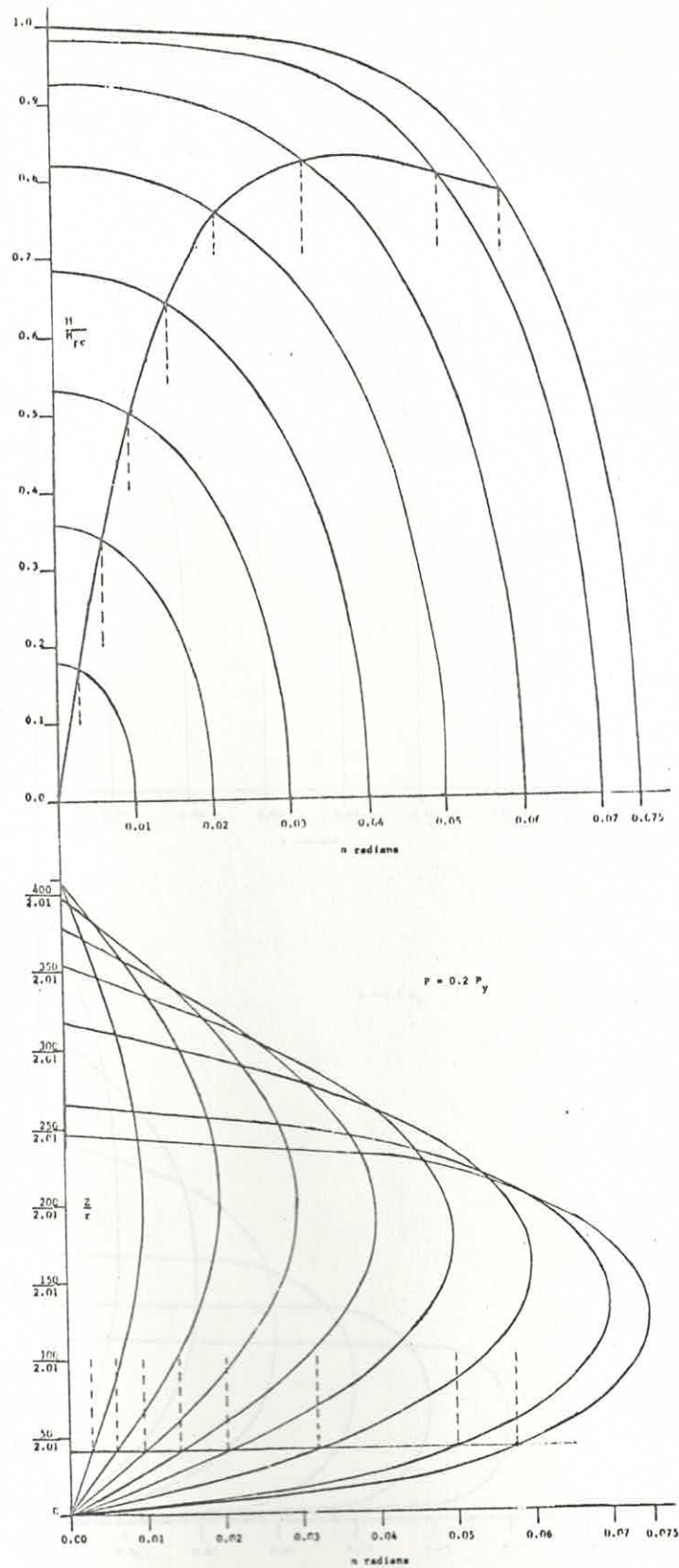


Fig. 6.-- M/M_{pc} vs. α and Z/r vs. α Curves

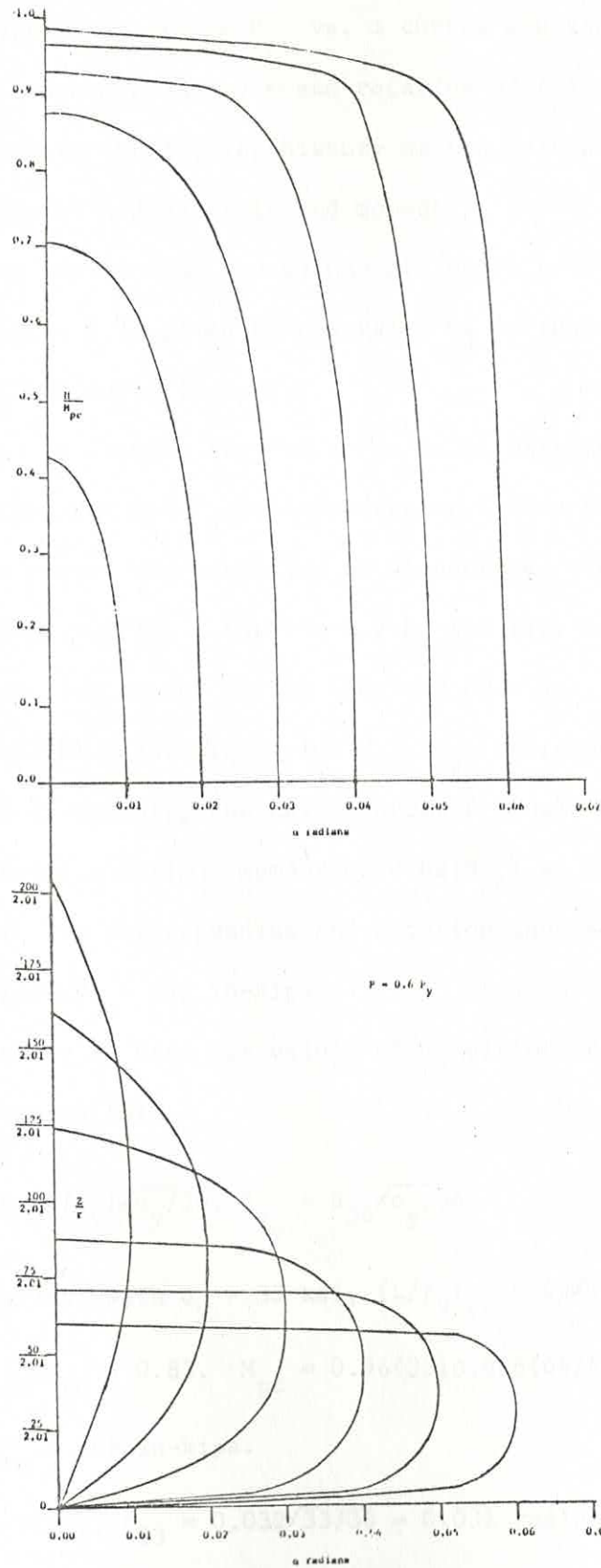


Fig. 7.-- M/M_{pc} vs. α and Z/r vs. α Curves

section are projected on to the M/M_{pc} vs. α curves and thus the M_0/M_{pc} vs. θ curve, where M_0 = end-moment and θ = end rotation of the column, is obtained. Such a curve represents the loading history of the column in the presence of a constant axial thrust and variable end moment.

The maximum moment that the column of length L can resist in the presence of the designated P is given by the value M_0 corresponding to M/M_{pc} at which the slope of the curve is zero.

The process of drawing the $M-\theta$ curve is illustrated in Fig. 6, the cardinal values being $P = 0.2 P_y$ and slenderness ratio = 40.

The column curves corresponding to slenderness ratios $L/r = 20, 30$, and 40 have been obtained. (Fig. 8 for $P = 0.2 P_y$ and Fig. 9 for $P = 0.6 P_y$)

An 8WF31 section pinned at the ends and 80.4 in., long has a slenderness ratio of $80.4/2.01 = 40$. The value M_{max}/M_{pc} corresponding to $L/r = 40$ is, from the curves, 0.82 assuming the axial thrust P equals $0.2 P_y$. The column can, therefore, resist a maximum moment of $0.82(M_{pc})$ at each end (symmetrical single curvature). The corresponding end rotation equals 0.034 radian.

$$M_{max} = 0.82(498.816)0.96 = 392 \text{ in-kips.}$$

The curves may be used for values of σ_y different from 36 ksi with the following modifications.(3)

$$(L/r_y)_{eq} = (L/r_y) \sqrt{\sigma_y/36}, \quad \theta_{\sigma_y} = \theta_{36} \sqrt{\sigma_y/36}.$$

As an illustration, with $\sigma_y = 33$ ksi, $(L/r_y)_{eq} = 40 \sqrt{33/36} = 38.3$.

Therefore, $M_{max}/M_{pc} = 0.83$. $M_{pc} = 0.96(33)0.866(64/4) = 439 \text{ in-k.}$

Therefore, $M_{max} = 366 \text{ in-kips.}$

$$\theta_{36} = 0.032. \quad \text{Therefore, } \theta_{33} = 0.032 \sqrt{33/36} = 0.031 \text{ radian.}$$

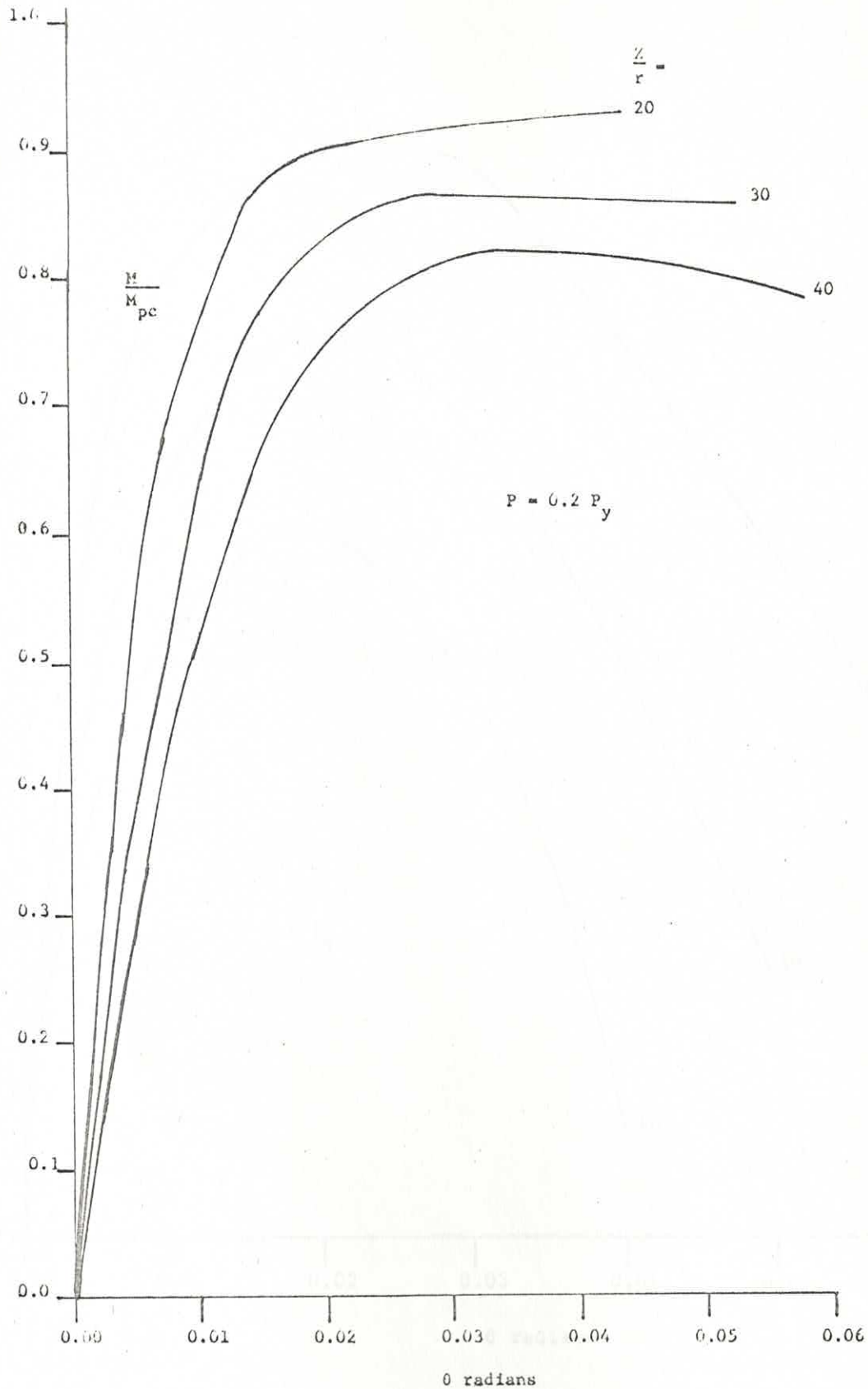


Fig. 8.-- M/M_{pc} vs. θ Curve for $P = 0.2P_y$

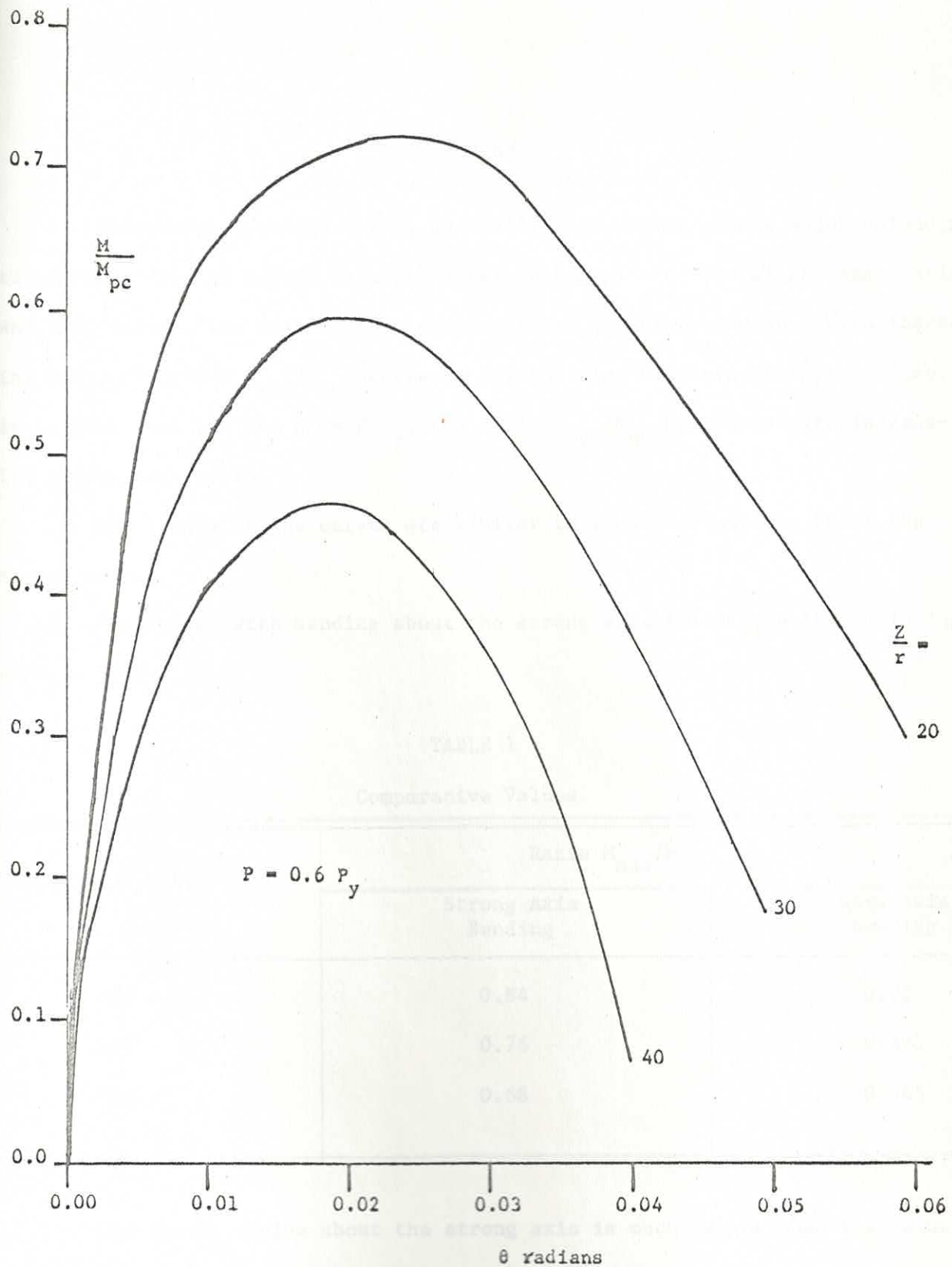


Fig. 9.-- M/M_{pc} vs. θ Curve for $P = 0.6 P_y$

SUMMARY

1. The moment-rotation curves generally form a peak beyond which unloading takes place and the column fails by local buckling. For low slenderness ratios and P/P_y values, the failure may be expected to be rather sudden. With increasing P/P_y , the ratio $M_{\max}/M_{pc}$ decreases for the same slenderness ratio. Also, it is true that for the same P/P_y , the ratio $M_{\max}/M_{pc}$ decreases with increasing slenderness ratio.

The shapes of the curves are similar to those for bending about the strong axis.

2. Comparison with bending about the strong axis (2) yields the following: ($P/P_y = 0.6$).

TABLE 1
Comparative Values

Slenderness Ratio	Ratio $M_{\max}/M_{pc}$	
	Strong Axis Bending	Weak Axis Bending
20	0.84	0.72
30	0.76	0.595
40	0.68	0.465

As the M_{pc} -value about the strong axis is much larger than its value about the weak axis, the column is considerably weaker when bending is about the weak axis.

3. Further work needs to be done in the following fields:

A. Extend CDC's over a wider range of P/P_y values;

B. Develop curves for different end conditions:

- a. One end fixed with moment less than M_{pc} at the other end,
- b. Double curvature with equal end moments,
- c. One end pinned,
- d. Variable ratios of end moments.

LITERATURE CITED

- (1) Fritz Engineering Laboratory, Department of Civil Engineering. Plastic Design of Multistory Frames. Vol. 1., Lehigh University, Bethlehem, Pennsylvania. Summer 1965.
- (2) Fritz Engineering Laboratory, Department of Civil Engineering. Plastic Design of Multistory Frames--Design Aids. Lehigh University, Bethlehem, Pennsylvania. Summer 1965.
- (3) Ketter, Robert L., Edmund L. Kaminsky, and Lynn S. Beedle. "Plastic Deformation of Wide-Flange Beam-Columns," Trans. ASCE. 120, New York, 1955, pp. 1028-1061.
- (4) Ojalvo, Morris. "Restrained Columns," Proc. ASCE., 86 No. EM5, New York, October 1960, pp. 1-11.

NOMENCLATURE FOR TEXT

APPENDIX A

NOMENCLATURE FOR TEXT

NOMENCLATURE FOR TEXT

- A = Area of cross section.
- CDC = Column Deflection Curve.
- E = Young's modulus = 30,000 ksi.
- L = Length of Column.
- M = Moment at a section.
- M_o = Moment applied at the end of a member (column).
- M_{mj} = Mean moment in jth segment.
- M_p = (Plastic) Moment capacity of the cross section.
- M_{pc} = (Plastic) Moment Capacity of the cross section in the presence of axial thrust.
- M_y = Moment at which yield point is reached in flexure.
- P = Axial thrust on column.
- P_y = Axial thrust corresponding to yield stress over the entire cross section.
- S_y = Section modulus of the section about the weak axis.
- Z = Distance from the center of a half-wave length of a CDC towards the ends.
- b = Flange width.
- d = Depth of wide flange section.
- r = Radius of gyration (about the weak axis).
- t = Flange thickness.
- v = Deflection from thrust line at the end of a segment of the CDC.
- w = Web thickness.

- α = Slope of the CDC at the end of a segment.
- ϵ = Strain.
- ρ = Subdivision along the thrust line of the segment of a CDC.
- σ = Normal stress.
- σ_{rc} = Maximum residual (normal) compressive stress.
- σ_{rt} = Maximum residual (normal) tensile stress.
- σ_y = Yield point stress (36 ksi unless specified otherwise).
- ϕ = Curvature at a section.
- ϕ_y = Curvature corresponding to initial outer-fiber yielding ($P = 0$).
- θ = Column end rotation.

APPENDIX B
NOMENCLATURE FOR COMPUTER ANALYSIS

NOMENCLATURE FOR COMPUTER ANALYSIS

$$MY = M_y$$

$$AO = \alpha_0$$

$$RHO = \rho$$

$$MM(J) = \text{Mean Moment } M_{mj}$$

$$RMM(J) = M_{mj}/M_y$$

$$RPHI(J) = \phi_j/\phi_y$$

$$A(J) = \alpha_j$$

$$V(J) = v_j$$

$$M(J) = M_j$$

$$MPC = M_{pc}$$

$$RATIO(J) = M_j/M_{pc}$$

$$Z(I) = \Sigma \rho_j$$

APPENDIX C

COMPUTER ANALYSIS

TABLE 2

Program for $P = 0.2P_y$ and $\alpha_0 = 0.01, 0.02, \dots$

```

DIMENSION MM(100),RMM(100),RPHI(100),A(100),V(100),M(100),
1RATIO(100),Z(100)
REAL MM,M,MY,MPC
P=C.2*36.C*9.12
MY=36.0*9.2
I=1
K=1
WRITE(6,1)
1 FORMAT(1H1,///,4X,1H1,4X,1HJ,5X,4HV(J),6X,6HRHU(J),5X,4HZ(J),
17X,6HRMM(J),5X,6HMJ/MPC,7X,4HA(J),7X,4HA(0),5X,7HRPHI(J))
35 F1=1
AO=0.01*F1
RHO=8.C
MM(1)=F*AO*RHO/2.0
RMM(1)=MM(1)/MY
RPHI(1)=RMM(1)
A(1)=AO-0.0003*RHO*RPHI(1)
V(1)=RHO*AO-C.00015*RPHI(1)*RHO**2
M(1)=P*V(1)
MPC=576.0*0.866*C.96
RATIO(1)=M(1)/MPC
WRITE(6,2) I,K,V(1),RHO,RHO,RMM(1),RATIO(1),A(1),AO,RPHI(1)
2 FORMAT(2I5,8F11.5)
J=2
Z(1)=RHO
34 RHO=8.C
32 MM(J)=M(J-1)+F*A(J-1)*RHO/2.0
RMM(J)=MM(J)/MY
IF(RMM(J) .LE. 0.653) GO TO 21
IF(RMM(J) .LE. 1.000) GO TO 22
IF(RMM(J) .LE. 1.150) GO TO 23
IF(RMM(J) .LE. 1.227) GO TO 24
IF(RMM(J) .LE. 1.286) GO TO 25
IF(RMM(J) .LE. 1.318) GO TO 26
IF(RMM(J) .LE. 1.355) GO TO 27
IF(RMM(J) .LE. 1.396) GO TO 40
IF(RMM(J) .LE. 1.441) GO TO 41
RPHI(J)=9.000*(RMM(J)-1.441)*4.000/0.049
GO TO 28
21 RPHI(J)=RMM(J)
GO TO 28
22 RPHI(J)=C.653*(RMM(J)-0.653)*C.606/0.347
GO TO 28
23 RPHI(J)=1.255*(RMM(J)-1.000)*0.446/0.150
GO TO 28
24 RPHI(J)=1.705*(RMM(J)-1.150)*C.295/0.077
GO TO 28
25 RPHI(J)=2.000*(RMM(J)-1.227)*0.402/0.059
GO TO 28
26 RPHI(J)=2.402*(RMM(J)-1.286)*0.598/0.032
GO TO 28
27 RPHI(J)=3.000*(RMM(J)-1.318)*1.000/0.037
GO TO 28
40 RPHI(J)=4.000*(RMM(J)-1.355)*2.000/C.041
GO TO 28

```

TABLE 2--Continued

```

41 RPHI(J)=C.COC+(RMM(J)-1.396)*3.000/0.045
28 A(J)=A(J-1)-0.0003*RHC*RPHI(J)
  IF(A(J).GE. 0.0) GO TO 29
  IF(RHO.EQ. 1.0 .AND. A(J).LT. 0.00) GO TO 29
  IF(A(J).LT. 0.00) RHO=1.0
  GO TO 32
29 V(J)=V(J-1)+A(J-1)*RHC-0.00015*RPHI(J)*RHC**2
  M(J)=P*V(J)
  RATIO(J)=M(J)/MPC
33 Z(I)=Z(I)+RHO
  IF(RATIO(J).GE. 1.0) GO TO 31
  IF(RHO.EQ. 1.0 .AND. A(J).GE. 0.0000) GO TO 36
  IF(A(J).LT. 0.0000) GO TO 30
36 WRITE(6,2) I,J,V(J),RHC,Z(I),RMM(J),RATIO(J),A(J),AQ,RPHI(J)
  J=J+1
  IF(RHO.EQ. 1.0) GO TO 32
  GO TO 34
30 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AQ,RPHI(J)
  I=I+1
  GO TO 35
31 CONTINUE
  RETURN
  END

```

TABLE 3

Results for $P = 0.2P_y$ and $\alpha_0 = 0.01, 0.02, \dots$

	J	V(J)	FBC(J)	Z(J)	RMM(J)	PJ/4PC	A(J)	A(G)	RPHI(J)
1	1	0.07952	8.00000	8.00000	0.00793	0.01096	0.00998	0.01000	0.00793
1	2	0.15954	8.00000	16.00000	0.02376	0.02188	0.00992	0.01000	0.02376
1	3	0.23856	8.00000	24.00000	0.03950	0.03271	0.00983	0.01000	0.03950
1	4	0.31666	8.00000	32.00000	0.05509	0.04342	0.00970	0.01000	0.05509
1	5	0.39356	8.00000	40.00000	0.07047	0.05397	0.00953	0.01000	0.07047
1	6	0.46856	8.00000	48.00000	0.08558	0.06431	0.00932	0.01000	0.08558
1	7	0.54258	8.00000	56.00000	0.10037	0.07440	0.00908	0.01000	0.10037
1	8	0.61413	8.00000	64.00000	0.11477	0.08421	0.00881	0.01000	0.11477
1	9	0.68334	8.00000	72.00000	0.12874	0.09370	0.00850	0.01000	0.12874
1	10	0.74955	8.00000	80.00000	0.14222	0.10284	0.00816	0.01000	0.14222
1	11	0.81371	8.00000	88.00000	0.15515	0.11158	0.00778	0.01000	0.15515
1	12	0.87436	8.00000	96.00000	0.16750	0.11990	0.00738	0.01000	0.16750
1	13	0.93170	8.00000	104.00000	0.17921	0.12776	0.00695	0.01000	0.17921
1	14	0.98548	8.00000	112.00000	0.19023	0.13513	0.00649	0.01000	0.19023
1	15	1.03551	8.00000	120.00000	0.20053	0.14199	0.00601	0.01000	0.20053
1	16	1.08160	8.00000	128.00000	0.21007	0.14831	0.00551	0.01000	0.21007
1	17	1.12358	8.00000	136.00000	0.21881	0.15407	0.00498	0.01000	0.21881
1	18	1.16127	8.00000	144.00000	0.22671	0.15924	0.00444	0.01000	0.22671
1	19	1.19455	8.00000	152.00000	0.23376	0.16360	0.00388	0.01000	0.23376
1	20	1.22328	8.00000	160.00000	0.23991	0.16774	0.00330	0.01000	0.23991
1	21	1.24735	8.00000	168.00000	0.24515	0.17104	0.00271	0.01000	0.24515
1	22	1.26667	8.00000	176.00000	0.24945	0.17369	0.00212	0.01000	0.24945
1	23	1.28118	8.00000	184.00000	0.25281	0.17568	0.00151	0.01000	0.25281
1	24	1.29080	8.00000	192.00000	0.25523	0.17700	0.00090	0.01000	0.25520
1	25	1.29552	8.00000	200.00000	0.25663	0.17765	0.00028	0.01000	0.25663
1	26	1.29576	1.00000	201.00000	0.25688	0.17768	0.00020	0.01000	0.25688
1	27	1.29592	1.00000	202.00000	0.25692	0.17770	0.00013	0.01000	0.25692
1	28	1.29601	1.00000	203.00000	0.25694	0.17772	0.00005	0.01000	0.25694
1	29	1.29602	1.00000	204.00000	0.25695	0.17772	-0.00003	0.01000	0.25695
2	1	0.15985	8.00000	8.00000	0.01586	0.02192	0.01996	0.02000	0.01586
2	2	0.31909	8.00000	16.00000	0.04752	0.04375	0.01985	0.02000	0.04752
2	3	0.47711	8.00000	24.00000	0.07900	0.06542	0.01966	0.02000	0.07900
2	4	0.63332	8.00000	32.00000	0.11018	0.08684	0.01939	0.02000	0.11018
2	5	0.78712	8.00000	40.00000	0.14094	0.10793	0.01906	0.02000	0.14094
2	6	0.93792	8.00000	48.00000	0.17117	0.12861	0.01864	0.02000	0.17117
2	7	1.08515	8.00000	56.00000	0.20074	0.14880	0.01816	0.02000	0.20074
2	8	1.22825	8.00000	64.00000	0.22955	0.16842	0.01761	0.02000	0.22955
2	9	1.36668	8.00000	72.00000	0.25748	0.18740	0.01699	0.02000	0.25748
2	10	1.49950	8.00000	80.00000	0.28444	0.20567	0.01631	0.02000	0.28444
2	11	1.62741	8.00000	88.00000	0.31031	0.22316	0.01557	0.02000	0.31031
2	12	1.74873	8.00000	96.00000	0.33500	0.23979	0.01476	0.02000	0.33500
2	13	1.86339	8.00000	104.00000	0.35841	0.25552	0.01390	0.02000	0.35841
2	14	1.97096	8.00000	112.00000	0.38046	0.27027	0.01299	0.02000	0.38046
2	15	2.07102	8.00000	120.00000	0.40107	0.28399	0.01203	0.02000	0.40107
2	16	2.16321	8.00000	128.00000	0.42014	0.29663	0.01102	0.02000	0.42014
2	17	2.24715	8.00000	136.00000	0.43762	0.30814	0.00997	0.02000	0.43762
2	18	2.32255	8.00000	144.00000	0.45343	0.31848	0.00888	0.02000	0.45343
2	19	2.38910	8.00000	152.00000	0.46751	0.32760	0.00776	0.02000	0.46751
2	20	2.44656	8.00000	160.00000	0.47982	0.33548	0.00661	0.02000	0.47982
2	21	2.49470	8.00000	168.00000	0.49030	0.34209	0.00543	0.02000	0.49030
2	22	2.53335	8.00000	176.00000	0.49891	0.34738	0.00423	0.02000	0.49891
2	23	2.56235	8.00000	184.00000	0.50562	0.35136	0.00302	0.02000	0.50562
2	24	2.58161	8.00000	192.00000	0.51041	0.35400	0.00179	0.02000	0.51041
2	25	2.59163	8.00000	200.00000	0.51325	0.35529	0.00056	0.02000	0.51325
2	26	2.59152	1.00000	201.00000	0.51376	0.35536	0.00041	0.02000	0.51376
2	27	2.59185	1.00000	202.00000	0.51384	0.35541	0.00025	0.02000	0.51384
2	28	2.59202	1.00000	203.00000	0.51389	0.35543	0.00010	0.02000	0.51389
2	29	2.59205	1.00000	204.00000	0.51391	0.35543	-0.00005	0.02000	0.51391

TABLE 3--Continued

I	J	V(J)	RHO(J)	Z(J)	RMM(J)	MJ/MPC	A(J)	A(O)	RPHI(J)
3	1	0.24577	8.00000	8.00000	0.32379	0.63286	0.02994	0.03000	0.02379
3	2	0.41865	8.00000	16.00000	0.07128	0.06563	0.02977	0.03000	0.07128
3	3	0.71567	8.00000	24.00000	0.11850	0.09814	0.02949	0.03000	0.11850
3	4	0.94948	8.00000	32.00000	0.16527	0.13027	0.02909	0.03000	0.16527
3	5	1.18063	8.00000	40.00000	0.21141	0.16190	0.02858	0.03000	0.21141
3	6	1.40688	8.00000	48.00000	0.25675	0.19292	0.02797	0.03000	0.25675
3	7	1.62775	8.00000	56.00000	0.30111	0.22320	0.02724	0.03000	0.30111
3	8	1.84238	8.00000	64.00000	0.34432	0.25264	0.02642	0.03000	0.34432
3	9	2.05000	8.00000	72.00000	0.38622	0.28111	0.02549	0.03000	0.38622
3	10	2.24985	8.00000	80.00000	0.42665	0.30851	0.02447	0.03000	0.42665
3	11	2.44112	8.00000	88.00000	0.46546	0.33474	0.02335	0.03000	0.46546
3	12	2.62309	8.00000	96.00000	0.50250	0.35969	0.02214	0.03000	0.50250
3	13	2.79505	8.00000	104.00000	0.53762	0.38328	0.02085	0.03000	0.53762
3	14	2.95644	8.00000	112.00000	0.57069	0.40540	0.01948	0.03000	0.57069
3	15	3.10654	8.00000	120.00000	0.60160	0.42598	0.01804	0.03000	0.60160
3	16	3.24481	8.00000	128.00000	0.63021	0.44494	0.01653	0.03000	0.63021
3	17	3.37070	8.00000	136.00000	0.65643	0.46221	0.01495	0.03000	0.65643
3	18	3.48355	8.00000	144.00000	0.68013	0.47768	0.01327	0.03000	0.70038
3	19	3.58260	8.00000	152.00000	0.70117	0.49126	0.01150	0.03000	0.73713
3	20	3.66715	8.00000	160.00000	0.71941	0.50286	0.00965	0.03000	0.76897
3	21	3.73675	8.00000	168.00000	0.73471	0.51240	0.00774	0.03000	0.79570
3	22	3.79084	8.00000	176.00000	0.74699	0.51982	0.00578	0.03000	0.81715
3	23	3.82908	8.00000	184.00000	0.75616	0.52506	0.00378	0.03000	0.83316
3	24	3.85122	8.00000	192.00000	0.76215	0.52810	0.00176	0.03000	0.84363
3	25	3.85285	1.00000	193.00000	0.76372	0.52832	0.00150	0.03000	0.84636
3	26	3.85422	1.00000	194.00000	0.76402	0.52851	0.00125	0.03000	0.84688
3	27	3.85535	1.00000	195.00000	0.76427	0.52866	0.00099	0.03000	0.84731
3	28	3.85621	1.00000	196.00000	0.76446	0.52878	0.00074	0.03000	0.84766
3	29	3.85682	1.00000	197.00000	0.76461	0.52887	0.00048	0.03000	0.84791
3	30	3.85718	1.00000	198.00000	0.76471	0.52891	0.00023	0.03000	0.84808
3	31	3.85728	1.00000	199.00000	0.76475	0.52893	-0.00002	0.03000	0.84816
4	1	0.31970	8.00000	8.00000	0.03172	0.04384	0.03992	0.04000	0.03172
4	2	0.63817	8.00000	16.00000	0.09504	0.08751	0.03970	0.04000	0.09504
4	3	0.95422	8.00000	24.00000	0.15801	0.13085	0.03932	0.04000	0.15801
4	4	1.26664	8.00000	32.00000	0.22036	0.17369	0.03879	0.04000	0.22036
4	5	1.57424	8.00000	40.00000	0.28189	0.21587	0.03811	0.04000	0.28189
4	6	1.87584	8.00000	48.00000	0.34233	0.25722	0.03729	0.04000	0.34233
4	7	2.17030	8.00000	56.00000	0.40148	0.29760	0.03633	0.04000	0.40148
4	8	2.45650	8.00000	64.00000	0.45909	0.33685	0.03522	0.04000	0.45909
4	9	2.73335	8.00000	72.00000	0.51496	0.37481	0.03399	0.04000	0.51496
4	10	2.99980	8.00000	80.00000	0.56887	0.41135	0.03262	0.04000	0.56887
4	11	3.25482	8.00000	88.00000	0.62061	0.44632	0.03113	0.04000	0.62061
4	12	3.49734	8.00000	96.00000	0.66999	0.47957	0.02950	0.04000	0.66999
4	13	3.72596	8.00000	104.00000	0.71678	0.51092	0.02766	0.04000	0.71678
4	14	3.93917	8.00000	112.00000	0.76065	0.54016	0.02564	0.04000	0.76065
4	15	4.13555	8.00000	120.00000	0.80132	0.56709	0.02345	0.04000	0.80132
4	16	4.31380	8.00000	128.00000	0.83852	0.59153	0.02111	0.04000	0.83852
4	17	4.47273	8.00000	136.00000	0.87200	0.61332	0.01862	0.04000	0.87200
4	18	4.61128	8.00000	144.00000	0.90154	0.63232	0.01601	0.04000	0.90154
4	19	4.72853	8.00000	152.00000	0.92694	0.64840	0.01330	0.04000	0.92694
4	20	4.82371	8.00000	160.00000	0.94803	0.66145	0.01050	0.04000	0.94803
4	21	4.89618	8.00000	168.00000	0.96468	0.67139	0.00762	0.04000	0.96468
4	22	4.94546	8.00000	176.00000	0.97677	0.67814	0.00470	0.04000	0.97677
4	23	4.97122	8.00000	184.00000	0.98422	0.68168	0.00174	0.04000	0.98422
4	24	4.97278	1.00000	185.00000	0.98577	0.68189	0.00137	0.04000	0.98577
4	25	4.97350	1.00000	186.00000	0.98603	0.68205	0.00100	0.04000	0.98603
4	26	4.97478	1.00000	187.00000	0.98624	0.68217	0.00063	0.04000	0.98624
4	27	4.97522	1.00000	188.00000	0.98637	0.68223	0.00026	0.04000	0.98637
4	28	4.97530	1.00000	189.00000	0.98642	0.68224	-0.00011	0.04000	0.98642
5	1	0.39962	8.00000	8.00000	0.03965	0.05480	0.04990	0.05000	0.03965
5	2	0.79772	8.00000	16.00000	0.11881	0.10939	0.04962	0.05000	0.11881
5	3	1.19278	8.00000	24.00000	0.19751	0.16356	0.04915	0.05000	0.19751
5	4	1.58330	8.00000	32.00000	0.27546	0.21711	0.04848	0.05000	0.27546
5	5	1.96779	8.00000	40.00000	0.35236	0.26993	0.04764	0.05000	0.35236
5	6	2.34480	8.00000	48.00000	0.42792	0.32153	0.04661	0.05000	0.42792
5	7	2.71288	8.00000	56.00000	0.50185	0.37200	0.04541	0.05000	0.50185

TABLE 3--Continued

I	J	V(J)	RHO(J)	Z(J)	RMM(J)	MJ/MPC	A(J)	A(O)	RPHI(J)
5	8	5.61665	8.00000	64.00000	0.57367	0.42106	0.04403	0.05000	0.57387
5	9	5.61665	8.00000	72.00000	0.64370	0.46851	0.04249	0.05000	0.64370
5	10	5.74933	8.00000	80.00000	0.71109	0.51413	0.04067	0.05000	0.75445
5	11	6.00660	8.00000	88.00000	0.77560	0.55760	0.03859	0.05000	0.86711
5	12	6.36580	8.00000	96.00000	0.83681	0.59866	0.03626	0.05000	0.97401
5	13	6.64553	8.00000	104.00000	0.89432	0.63702	0.03368	0.05000	1.07444
5	14	6.90374	8.00000	112.00000	0.94773	0.67242	0.03087	0.05000	1.16777
5	15	7.13871	8.00000	120.00000	0.99670	0.70464	0.02787	0.05000	1.25325
5	16	7.34835	8.00000	128.00000	1.04090	0.73340	0.02455	0.05000	1.33062
5	17	7.53045	8.00000	136.00000	1.07965	0.75836	0.02096	0.05000	1.49642
5	18	7.68283	8.00000	144.00000	1.11310	0.77926	0.01713	0.05000	1.59527
5	19	7.80381	8.00000	152.00000	1.14027	0.79585	0.01311	0.05000	1.67607
5	20	7.89192	8.00000	160.00000	1.16167	0.80793	0.00892	0.05000	1.74740
5	21	7.94556	8.00000	168.00000	1.17521	0.81534	0.00459	0.05000	1.80158
5	22	7.96515	8.00000	176.00000	1.18249	0.81797	0.00020	0.05000	1.82949
5	23	7.96508	8.00000	177.00000	1.18268	0.81796	-0.00035	0.05000	1.83018
6	1	0.47954	8.00000	8.00000	0.04758	0.05576	0.05989	0.06000	0.04758
6	2	0.55726	8.00000	16.00000	0.14257	0.13126	0.05954	0.06000	0.14257
6	3	1.41133	8.00000	24.00000	0.25701	0.19627	0.05897	0.06000	0.25701
6	4	1.89996	8.00000	32.00000	0.33055	0.26153	0.05818	0.06000	0.33055
6	5	2.36135	8.00000	40.00000	0.42283	0.32380	0.05717	0.06000	0.42283
6	6	2.81376	8.00000	48.00000	0.51350	0.38584	0.05593	0.06000	0.51350
6	7	3.25545	8.00000	56.00000	0.60222	0.44640	0.05449	0.06000	0.60222
6	8	3.68450	8.00000	64.00000	0.68864	0.50524	0.05277	0.06000	0.71524
6	9	4.09841	8.00000	72.00000	0.77234	0.56199	0.05071	0.06000	0.86142
6	10	4.49443	8.00000	80.00000	0.85276	0.61630	0.04830	0.06000	1.00187
6	11	4.86593	8.00000	88.00000	0.92937	0.66779	0.04557	0.06000	1.13566
6	12	5.22240	8.00000	96.00000	1.00166	0.71612	0.04254	0.06000	1.26393
6	13	5.54867	8.00000	104.00000	1.06913	0.76086	0.03903	0.06000	1.46456
6	14	5.84505	8.00000	112.00000	1.13103	0.80150	0.03507	0.06000	1.64861
6	15	6.10790	8.00000	120.00000	1.18666	0.83754	0.03064	0.06000	1.84544
6	16	6.33328	8.00000	128.00000	1.23526	0.86845	0.02571	0.06000	2.05626
6	17	6.51652	8.00000	136.00000	1.27603	0.89358	0.02010	0.06000	2.33406
6	18	6.65037	8.00000	144.00000	1.30791	0.91193	0.01336	0.06000	2.61154
6	19	6.72554	8.00000	152.00000	1.32910	0.92224	0.00544	0.06000	3.29998
6	20	6.73046	1.00000	153.00000	1.33395	0.92291	0.00441	0.06000	3.43107
6	21	6.73435	1.00000	154.00000	1.33482	0.92345	0.00337	0.06000	3.45469
6	22	6.73720	1.00000	155.00000	1.33549	0.92384	0.00233	0.06000	3.47275
6	23	6.73900	1.00000	156.00000	1.33595	0.92408	0.00128	0.06000	3.48523
6	24	6.73976	1.00000	157.00000	1.33621	0.92419	0.00024	0.06000	3.49210
6	25	6.73947	1.00000	158.00000	1.33625	0.92415	-0.00081	0.06000	3.49336
7	1	0.55547	8.00000	8.00000	0.05551	0.07672	0.06987	0.07000	0.05551
7	2	1.11680	8.00000	16.00000	0.16633	0.15314	0.06947	0.07000	0.16633
7	3	1.66989	8.00000	24.00000	0.27651	0.22898	0.06880	0.07000	0.27651
7	4	2.21662	8.00000	32.00000	0.38564	0.30395	0.06788	0.07000	0.38564
7	5	2.75491	8.00000	40.00000	0.49330	0.37777	0.06669	0.07000	0.49330
7	6	3.28272	8.00000	48.00000	0.59908	0.45014	0.06526	0.07000	0.59908
7	7	3.79167	8.00000	56.00000	0.70259	0.52075	0.06348	0.07000	0.73960
7	8	4.29674	8.00000	64.00000	0.80327	0.58919	0.06128	0.07000	0.91544
7	9	4.77659	8.00000	72.00000	0.90048	0.65499	0.05868	0.07000	1.08519
7	10	5.23406	8.00000	80.00000	0.99355	0.71772	0.05569	0.07000	1.24773
7	11	5.66512	8.00000	88.00000	1.08187	0.77683	0.05208	0.07000	1.50243
7	12	6.06486	8.00000	96.00000	1.16447	0.83164	0.04785	0.07000	1.76045
7	13	6.42762	8.00000	104.00000	1.24037	0.88139	0.04284	0.07000	2.09113
7	14	6.74324	8.00000	112.00000	1.30832	0.92467	0.03607	0.07000	2.51903
7	15	6.98848	8.00000	120.00000	1.36553	0.95829	0.02524	0.07000	4.51351
7	16	7.12666	8.00000	128.00000	1.40556	0.97724	0.00931	0.07000	6.63709
7	17	7.13490	1.00000	129.00000	1.41386	0.97837	0.00715	0.07000	7.19077
7	18	7.14095	1.00000	130.00000	1.41528	0.97920	0.00497	0.07000	7.28529
7	19	7.14482	1.00000	131.00000	1.41626	0.97973	0.00276	0.07000	7.35093
7	20	7.14647	1.00000	132.00000	1.41681	0.97996	0.00054	0.07000	7.38742
7	21	7.14591	1.00000	133.00000	1.41692	0.97988	-0.00167	0.07000	7.39462
8	1	0.63935	8.00000	8.00000	0.06344	0.08768	0.07985	0.08000	0.06344
8	2	1.27635	8.00000	16.00000	0.19009	0.17502	0.07939	0.08000	0.19009
8	3	1.90845	8.00000	24.00000	0.31601	0.26170	0.07863	0.08000	0.31601
8	4	2.53328	8.00000	32.00000	0.44073	0.34738	0.07758	0.08000	0.44073

TABLE 3--Continued

I	J	V(J)	RHO(J)	Z(J)	RMM(J)	MJ/MPC	A(J)	A(O)	RPHI(J)
8	5	3.14847	8.00000	40.00000	0.55377	0.43173	0.07622	0.08000	0.56377
8	6	3.75145	8.00000	48.00000	0.68467	0.51442	0.07452	0.08000	0.70830
8	7	4.34885	8.00000	56.00000	0.80287	0.59496	0.07233	0.08000	0.91472
8	8	4.90676	8.00000	64.00000	0.91758	0.67284	0.06965	0.08000	1.11507
8	9	5.45108	8.00000	72.00000	1.02805	0.74748	0.06643	0.08000	1.34241
8	10	5.96662	8.00000	80.00000	1.13342	0.81817	0.06246	0.08000	1.65569
8	11	6.44670	8.00000	88.00000	1.23248	0.88400	0.05757	0.08000	2.03731
8	12	6.87693	8.00000	96.00000	1.32373	0.94300	0.04999	0.08000	3.15624
8	13	7.21473	8.00000	104.00000	1.40307	0.98932	0.03446	0.08000	6.47138

TABLE 4

Program for $P = 0.2P_y$ and $\alpha_0 = 0.075$

```

    DIMENSION MM(100),RMM(100),RPHI(100),A(100),V(100),M(100),
1  RATIO(100),Z(100)
    REAL MM,M,MY,MPC
    P=0.2*36.0*9.12
    MY=36.0*9.2
    I=8
    K=1
    WRITE(6,1)
1  FORMAT(4X,1HI,4X,1HJ,5X,4HV(J),6X,6HRHO(J),5X,4HZ(J),7X,6HRMM(J),
15X,6HNMJ/MPC,7X,4HA(J),7X,4HA(0),5X,7HRPHI(J))
    FI=7.5
    AO=0.01*FI
    RHO=8.0
    MM(1)=P*AO*RHO/2.0
    RMM(1)=MM(1)/MY
    RPHI(1)=RMM(1)
    A(1)=AO-C.0003*RHO*RPHI(1)
    V(1)=RHO*AO-0.00015*RPHI(1)*RHO**2
    M(1)=P*V(1)
    MPC=576.0*0.866*0.96
    RATIO(1)=M(1)/MPC
    WRITE(6,2) I,K,V(1),RHO,RHO,RMM(1),RATIO(1),A(1),AO,RPHI(1)
2  FORMAT(2I5,8F11.5)
    J=2
    Z(1)=RHO
34 RHO=8.0
32 MM(J)=M(J-1)+P*A(J-1)*RHO/2.0
    RMM(J)=MM(J)/MY
    IF(RMM(J) .LE. 0.653) GO TO 21
    IF(RMM(J) .LE. 1.000) GO TO 22
    IF(RMM(J) .LE. 1.150) GO TO 23
    IF(RMM(J) .LE. 1.227) GO TO 24
    IF(RMM(J) .LE. 1.286) GO TO 25
    IF(RMM(J) .LE. 1.318) GO TO 26
    IF(RMM(J) .LE. 1.355) GO TO 27
    IF(RMM(J) .LE. 1.396) GO TO 40
    IF(RMM(J) .LE. 1.441) GO TO 41
    RPHI(J)=9.000+(RMM(J)-1.441)*4.000/0.049
    GO TO 28
21 RPHI(J)=RMM(J)
    GO TO 28
22 RPHI(J)=0.653+(RMM(J)-0.653)*0.606/0.347
    GO TO 28
23 RPHI(J)=1.259+(RMM(J)-1.000)*0.446/0.150
    GO TO 28
24 RPHI(J)=1.705+(RMM(J)-1.150)*0.295/0.077
    GO TO 28
25 RPHI(J)=2.000+(RMM(J)-1.227)*0.402/0.059
    GO TO 28
26 RPHI(J)=2.402+(RMM(J)-1.286)*0.598/0.032
    GO TO 28
27 RPHI(J)=3.000+(RMM(J)-1.318)*1.000/0.037
    GO TO 28
40 RPHI(J)=4.000+(RMM(J)-1.355)*2.000/0.041
    GO TO 28

```

TABLE 4--Continued

```

41 RPHI(J)=c.000+(RMM(J)-1.3961)*3.000/0.045
28 A(J)=A(J-1)-0.0003*RHO*RPHI(J)
   IF(A(J) .GE. 0.0) GO TO 29
   IF(RHO .EQ. 1.0 .AND. A(J) .LT. 0.00) GO TO 29
   IF(A(J) .LT. 0.00) RHO=1.0
   GO TO 32
29 V(J)=V(J-1)+A(J-1)*RHC-0.00015*RPHI(J)*RHO**2
   M(J)=P*V(J)
   RATIO(J)=M(J)/MPC
33 Z(I)=Z(I)+RHO
   IF(RATIO(J) .GE. 1.0) GO TO 30
   IF(RHO .EQ. 1.0 .AND. A(J) .GE. 0.0000) GO TO 36
   IF(A(J) .LT. 0.0000) GO TO 30
36 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AO,RPHI(J)
   J=J+1
   IF(RHO .EQ. 1.0) GO TO 32
   GO TO 34
30 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AO,RPHI(J)
31 CONTINUE
   RETURN
   END

```

TABLE 5

Results for $P = 0.2P_y$ and $\alpha_0 = 0.075$

I	J	V(J)	RHO(J)	Z(J)	RMM(J)	MJ/MPG	A(J)	A(U)	RPHI(J)
8	1	0.59943	8.00000	8.00000	0.05948	0.08220	0.07486	0.07500	0.05948
8	2	1.19658	8.00000	16.00000	0.17821	0.16408	0.07443	0.07500	0.17821
8	3	1.78917	8.00000	24.00000	0.29626	0.24534	0.07372	0.07500	0.29626
8	4	2.37495	8.00000	32.00000	0.41318	0.32566	0.07273	0.07500	0.41318
8	5	2.95169	8.00000	40.00000	0.52854	0.40475	0.07146	0.07500	0.52854
8	6	3.51720	8.00000	48.00000	0.64187	0.48229	0.06992	0.07500	0.64187
8	7	4.06860	8.00000	56.00000	0.75277	0.55791	0.06793	0.07500	0.82724
8	8	4.60231	8.00000	64.00000	0.86052	0.63109	0.06550	0.07500	1.01541
8	9	5.11474	8.00000	72.00000	0.96440	0.70136	0.06262	0.07500	1.19683
8	10	5.60186	8.00000	80.00000	1.06372	0.76815	0.05915	0.07500	1.44847
8	11	6.05839	8.00000	88.00000	1.15754	0.83076	0.05499	0.07500	1.73387
8	12	6.47792	8.00000	96.00000	1.24475	0.88828	0.04990	0.07500	2.12093
8	13	6.84675	8.00000	104.00000	1.32389	0.93886	0.04231	0.07500	3.15910
8	14	7.13000	8.00000	112.00000	1.39100	0.97770	0.02850	0.07500	5.75607
8	15	7.27466	8.00000	120.00000	1.43620	0.99754	0.00767	0.07500	8.68008
8	16	7.28096	1.00000	121.00000	1.44304	0.99840	0.00492	0.07500	9.16663
8	17	7.28449	1.00000	122.00000	1.44402	0.99888	0.00214	0.07500	9.24620
8	18	7.28524	1.00000	123.00000	1.44444	0.99899	-0.00064	0.07500	9.28088

TABLE 6

Program for $P = 0.6P_y$ and $\alpha_0 = 0.01, 0.02, \dots$

```

DIMENSION MM(100),RMM(100),RPHI(100),A(100),V(100),M(100),
IRATIO(100),Z(100)
REAL MN,M,MY,MPC
P=0.6*36.C*9.12
MY=36.C*9.2
I=1
K=1
WRITE(6,1)
1  FORMAT(1H1,////,4X,1H1,4X,1HJ,5X,4HV(J),6X,6HRHO(J),5X,4HZ(J),
17X,6HRMM(J),5X,6HMJ/MPC,7X,4HA(J),7X,4HA(0),5X,7HRPHI(J))
35 FI=1
AO=C.C1*FI
RHO=8.C
MM(1)=P*AO*RHO/2.0
RMM(1)=MM(1)/MY
RPHI(1)=RMM(1)
A(1)=AO-0.0003*RHO*RPHI(1)
V(1)=RHO*AO-0.00015*RPHI(1)*RHO**2
M(1)=P*V(1)
MPC=576.0*C.C*66*C.C4
RATIO(1)=M(1)/MPC
WRITE(6,2) I,K,V(1),RHO,RHO,RMM(1),RATIO(1),A(1),AO,RPHI(1)
2  FORMAT(2I5,8F11.5)
J=2
Z(1)=RHO
34 RHO=8.C
32 MM(J)=N(J-1)+P*A(J-1)*RHO/2.0
RMM(J)=MM(J)/MY
IF(RMM(J) .LE. 12.75/55.00) GO TO 21
IF(RMM(J) .LE. 22.00/55.00) GO TO 22
IF(RMM(J) .LE. 29.75/55.00) GO TO 23
IF(RMM(J) .LE. 34.50/55.00) GO TO 24
IF(RMM(J) .LE. 38.90/55.00) GO TO 25
IF(RMM(J) .LE. 41.25/55.00) GO TO 26
IF(RMM(J) .LE. 43.00/55.00) GO TO 27
IF(RMM(J) .LE. 45.00/55.00) GO TO 40
IF(RMM(J) .LE. 46.00/55.00) GO TO 41
IF(RMM(J) .LE. 46.50/55.00) GO TO 42
IF(RMM(J) .LE. 46.75/55.00) GO TO 43
RPHI(J)=78.40/11.20+(RMM(J)-46.75/55.00)*440.0
GO TO 28
21 RPHI(J)=RMM(J)
GO TO 28
22 RPHI(J)=2.60/11.20+(RMM(J)-12.75/55.00)*(3.65/11.20)/(9.25/55.00)
GO TO 28
23 RPHI(J)=6.25/11.20+(RMM(J)-22.00/55.00)*(4.95/11.20)/(7.75/55.00)
GO TO 28
24 RPHI(J)=11.20/11.20+(RMM(J)-29.75/55.00)*(5.00/11.20)/(4.75/55.00)
GO TO 28
25 RPHI(J)=16.20/11.20+(RMM(J)-34.50/55.00)*(6.20/11.20)/(4.40/55.00)
GO TO 28
26 RPHI(J)=22.40/11.20+(RMM(J)-38.90/55.00)*(5.75/11.20)/(2.35/55.00)
GO TO 28
27 RPHI(J)=28.15/11.20+(RMM(J)-41.25/55.00)*(5.45/11.20)/(1.75/55.00)
GO TO 28

```

TABLE 6--Continued

```

40 RPHI(J)=33.60/11.20*(RMM(J)-43.00/55.00)*27.50
   GO TO 28
41 RPHI(J)=44.80/11.20*(RMM(J)-45.00/55.00)*55.0
   GO TO 28
42 RPHI(J)=56.00/11.20*(RMM(J)-46.00/55.00)*110.0
   GO TO 28
43 RPHI(J)=67.20/11.20*(RMM(J)-46.50/55.00)*220.0
28 A(J)=A(J-1)-0.0003*RHO*RPHI(J)
   IF(A(J) .GE. 0.0) GO TO 29
   IF(RHO .EQ. 1.0 .AND. A(J) .LT. 0.00) GO TO 29
   IF(A(J) .LT. 0.00) RHO=1.0
   GO TO 32
29 V(J)=V(J-1)+A(J-1)*RHO-0.00015*RPHI(J)*RHO**2
   M(J)=P*V(J)
   RATIO(J)=M(J)/MPC
33 Z(I)=Z(I)+RHO
   IF(RATIO(J) .GE. 1.0) GO TO 31
   IF(RHO .EQ. 1.0 .AND. A(J) .GE. 0.0000) GO TO 36
   IF(A(J) .LT. 0.0000) GO TO 30
36 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AO,RPHI(J)
   J=J+1
   IF(RHO .EQ. 1.0) GO TO 32
   GO TO 34
30 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AO,RPHI(J)
   I=I+1
   GO TO 35
31 CONTINUE
   RETURN
   END

```

TABLE 7

Results for $P = 0.6P_y$ and $\alpha_0 = 0.01, 0.02, \dots$

1	J	V(J)	RHC(J)	Z(J)	RMM(J)	MJ/MPC	A(J)	A(0)	RPHI(J)
1	1	0.07577	8.00000	8.00000	0.02379	0.04922	0.00994	0.01000	0.02379
1	2	0.15863	8.00000	16.00000	0.07110	0.09789	0.00977	0.01000	0.07110
1	3	0.23568	8.00000	24.00000	0.11760	0.14543	0.00949	0.01000	0.11760
1	4	0.31004	8.00000	32.00000	0.16276	0.19131	0.00910	0.01000	0.16276
1	5	0.38086	8.00000	40.00000	0.20605	0.23501	0.00860	0.01000	0.20605
1	6	0.44718	8.00000	48.00000	0.24700	0.27594	0.00798	0.01000	0.24718
1	7	0.50778	8.00000	56.00000	0.28496	0.31333	0.00717	0.01000	0.31333
1	8	0.56131	8.00000	64.00000	0.31909	0.34637	0.00621	0.01000	0.40125
1	9	0.60659	8.00000	72.00000	0.34863	0.37430	0.00511	0.01000	0.45850
1	10	0.64261	8.00000	80.00000	0.37295	0.39653	0.00390	0.01000	0.50561
1	11	0.66858	8.00000	88.00000	0.39148	0.41256	0.00260	0.01000	0.54154
1	12	0.68388	8.00000	96.00000	0.40384	0.42200	0.00123	0.01000	0.57008
1	13	0.68502	1.00000	97.00000	0.40713	0.42270	0.00105	0.01000	0.58039
1	14	0.68599	1.00000	98.00000	0.40775	0.42330	0.00088	0.01000	0.58235
1	15	0.68678	1.00000	99.00000	0.40828	0.42379	0.00070	0.01000	0.58399
1	16	0.68740	1.00000	100.00000	0.40869	0.42417	0.00053	0.01000	0.58531
1	17	0.68784	1.00000	101.00000	0.40901	0.42444	0.00035	0.01000	0.58629
1	18	0.68810	1.00000	102.00000	0.40922	0.42460	0.00018	0.01000	0.58695
1	19	0.68819	1.00000	103.00000	0.40932	0.42466	0.00000	0.01000	0.58728
1	20	0.68810	1.00000	104.00000	0.40932	0.42460	-0.00018	0.01000	0.58728
2	1	0.15554	8.00000	8.00000	0.04758	0.09845	0.01989	0.02000	0.04758
2	2	0.31726	8.00000	16.00000	0.14220	0.19577	0.01954	0.02000	0.14220
2	3	0.47133	8.00000	24.00000	0.23520	0.29084	0.01897	0.02000	0.23870
2	4	0.61913	8.00000	32.00000	0.32547	0.38204	0.01798	0.02000	0.41362
2	5	0.75727	8.00000	40.00000	0.41102	0.46728	0.01656	0.02000	0.59261
2	6	0.88167	8.00000	48.00000	0.48980	0.54404	0.01454	0.02000	0.83971
2	7	0.98750	8.00000	56.00000	0.55900	0.60935	0.01192	0.02000	1.09349
2	8	1.06952	8.00000	64.00000	0.61570	0.65996	0.00859	0.02000	1.38660
2	9	1.12241	8.00000	72.00000	0.65657	0.69259	0.00463	0.02000	1.64915
2	10	1.14216	8.00000	80.00000	0.67861	0.70478	0.00031	0.02000	1.80163
2	11	1.14220	1.00000	81.00000	0.67943	0.70480	-0.00023	0.02000	1.80732
3	1	0.23931	8.00000	8.00000	0.07137	0.14767	0.02983	0.03000	0.07137
3	2	0.47590	8.00000	16.00000	0.21331	0.29366	0.02932	0.03000	0.21331
3	3	0.70595	8.00000	24.00000	0.35280	0.43562	0.02820	0.03000	0.46658
3	4	0.92355	8.00000	32.00000	0.48697	0.56989	0.02620	0.03000	0.83083
3	5	1.12006	8.00000	40.00000	0.61165	0.69115	0.02293	0.03000	1.36569
3	6	1.28271	8.00000	48.00000	0.72074	0.79151	0.01774	0.03000	2.16179
3	7	1.38966	8.00000	56.00000	0.80513	0.85750	0.00900	0.03000	3.64121
3	8	1.39796	1.00000	57.00000	0.82922	0.86263	0.00762	0.03000	4.60703
3	9	1.40485	1.00000	58.00000	0.83375	0.86688	0.00616	0.03000	4.85617
3	10	1.41024	1.00000	59.00000	0.83741	0.87021	0.00462	0.03000	5.11532
3	11	1.41405	1.00000	60.00000	0.84016	0.87256	0.00300	0.03000	5.41788
3	12	1.41621	1.00000	61.00000	0.84195	0.87389	0.00131	0.03000	5.61411
3	13	1.41667	1.00000	62.00000	0.84273	0.87417	-0.00040	0.03000	5.70014
4	1	0.31905	8.00000	8.00000	0.09517	0.19690	0.03977	0.04000	0.09517
4	2	0.63405	8.00000	16.00000	0.28441	0.39125	0.03897	0.04000	0.33405
4	3	0.93835	8.00000	24.00000	0.46984	0.57902	0.03710	0.04000	0.77708
4	4	1.22003	8.00000	32.00000	0.64639	0.75284	0.03332	0.04000	1.57873
4	5	1.45166	8.00000	40.00000	0.80492	0.89577	0.02459	0.04000	3.63525
4	6	1.47384	1.00000	41.00000	0.87074	0.90945	0.01975	0.04000	16.12451
4	7	1.49040	1.00000	42.00000	0.88249	0.91967	0.01337	0.04000	21.29420
4	8	1.50004	1.00000	43.00000	0.89044	0.92562	0.00593	0.04000	24.79206
4	9	1.50202	1.00000	44.00000	0.89396	0.92684	-0.00197	0.04000	26.34346
5	1	0.39886	8.00000	8.00000	0.11896	0.24612	0.04971	0.05000	0.11896
5	2	0.79204	8.00000	16.00000	0.35551	0.48874	0.04858	0.05000	0.47183
5	3	1.16883	8.00000	24.00000	0.58668	0.72124	0.04561	0.05000	1.23658
5	4	1.49916	8.00000	32.00000	0.80372	0.92507	0.03697	0.05000	3.60236
5	5	1.53160	1.00000	33.00000	0.90267	0.94509	0.02792	0.05000	30.17464

TABLE 7--Continued

I	J	V(J)	RHO(J)	Z(J)	RMM(J)	MJ/MPC	A(J)	A(O)	RPHI(J)
5	6	1.55350	1.00000	34.00000	0.91927	0.95885	0.01667	0.05000	37.48045
5	7	1.56429	1.00000	35.00000	0.92919	0.96527	0.00412	0.05000	41.84361
5	8	1.56198	1.00000	36.00000	0.93164	0.96383	-0.00876	0.05000	42.92158
6	1	0.47863	8.00000	8.00000	0.14275	0.29534	0.05966	0.06000	0.14275
6	2	0.94973	8.00000	16.00000	0.42661	0.58604	0.05812	0.06000	0.64151
6	3	1.39575	8.00000	24.00000	0.70315	0.86126	0.05339	0.06000	1.97149
6	4	1.44821	1.00000	25.00000	0.84604	0.89364	0.05155	0.06000	6.12928
6	5	1.49895	1.00000	26.00000	0.87670	0.92371	0.04592	0.06000	18.74875
6	6	1.53826	1.00000	27.00000	0.90402	0.94920	0.03669	0.06000	30.76694
6	7	1.56889	1.00000	28.00000	0.92584	0.96810	0.02458	0.06000	40.36957
6	8	1.58645	1.00000	29.00000	0.94046	0.97894	0.01054	0.06000	46.80275
6	9	1.58556	1.00000	30.00000	0.94673	0.98086	-0.00433	0.06000	49.56137
7	1	0.55840	8.00000	8.00000	0.16654	0.34457	0.06960	0.07000	0.16654
7	2	1.10690	8.00000	16.00000	0.49772	0.68303	0.06753	0.07000	0.86452
7	3	1.60827	8.00000	24.00000	0.81902	0.99240	0.05781	0.07000	4.04606

TABLE 8

Program for $P = 0.6P_y$ and $\alpha_0 = 0.065$

```

DIMENSION MM(100),RMM(100),RPHI(100),A(100),V(100),M(100),
1RATIO(100),Z(100)
REAL MM,M,MY,MPC
P=0.6*36.0*9.12
MY=36.0*9.2
I=7
K=1
WRITE(6,1)
1 FORMAT(4X,1HI,4X,1HJ,5X,4HV(J),6X,6HRHO(J),5X,4HZ(J),7X,6HRMM(J),
15X,6HMJ/MPC,7X,4HA(J),7X,4HA(0),5X,7HRPHI(J))
F1=6.5
AO=0.01*F1
RHO=8.0
MM(1)=P*AO*RHO/2.0
RMM(1)=MM(1)/MY
RPHI(1)=RMM(1)
A(1)=AO-0.0003*RHO*RPHI(1)
V(1)=RHO*AO-0.00015*RPHI(1)*RHO**2
M(1)=P*V(1)
MPC=576.0*0.866*0.64
RATIO(1)=M(1)/MPC
WRITE(6,2) I,K,V(1),RHO,RHO,RMM(1),RATIO(1),A(1),AO,RPHI(1)
2 FORMAT(2I5,8F11.5)
J=2
Z(1)=RHO
34 RHO=8.0
32 MM(J)=M(J-1)+P*A(J-1)*RHO/2.0
RMM(J)=MM(J)/MY
IF(RMM(J) .LE. 12.75/55.00) GO TO 21
IF(RMM(J) .LE. 22.00/55.00) GO TO 22
IF(RMM(J) .LE. 29.75/55.00) GO TO 23
IF(RMM(J) .LE. 34.50/55.00) GO TO 24
IF(RMM(J) .LE. 38.90/55.00) GO TO 25
IF(RMM(J) .LE. 41.25/55.00) GO TO 26
IF(RMM(J) .LE. 43.00/55.00) GO TO 27
IF(RMM(J) .LE. 45.00/55.00) GO TO 40
IF(RMM(J) .LE. 46.00/55.00) GO TO 41
IF(RMM(J) .LE. 46.50/55.00) GO TO 42
IF(RMM(J) .LE. 46.75/55.00) GO TO 43
RPHI(J)=78.40/11.20+(RMM(J)-46.75/55.00)*440.0
GO TO 28
21 RPHI(J)=RMM(J)
GO TO 28
22 RPHI(J)=2.60/11.20+(RMM(J)-12.75/55.00)*(3.65/11.20)/(9.25/55.00)
GO TO 28
23 RPHI(J)=6.25/11.20+(RMM(J)-22.00/55.00)*(4.95/11.20)/(7.75/55.00)
GO TO 28
24 RPHI(J)=11.20/11.20+(RMM(J)-29.75/55.00)*(5.00/11.20)/(4.75/55.00)
GO TO 28
25 RPHI(J)=16.20/11.20+(RMM(J)-34.50/55.00)*(6.20/11.20)/(4.40/55.00)
GO TO 28
26 RPHI(J)=22.40/11.20+(RMM(J)-38.90/55.00)*(5.75/11.20)/(2.35/55.00)
GO TO 28
27 RPHI(J)=28.15/11.20+(RMM(J)-41.25/55.00)*(5.45/11.20)/(1.75/55.00)
GO TO 28

```

TABLE 8--Continued

```

40 RPHI(J)=33.60/11.20+(RMM(J)-43.00/55.00)*27.50
   GU TO 28
41 RPHI(J)=44.80/11.20+(RMM(J)-45.00/55.00)*55.0
   GO TO 28
42 RPHI(J)=56.00/11.20+(RMM(J)-46.00/55.00)*110.0
   GO TO 28
43 RPHI(J)=67.20/11.20+(RMM(J)-46.50/55.00)*220.0
28 A(J)=A(J-1)-0.0003*RHO*RPHI(J)
   IF(A(J) .GE. 0.0) GO TO 29
   IF(RHO .EQ. 1.0 .AND. A(J) .LT. 0.00) GO TO 29
   IF(A(J) .LT. 0.00) RHO=1.0
   GO TO 32
29 V(J)=V(J-1)+A(J-1)*RHO-0.00015*RPHI(J)*RHO**2
   M(J)=P*V(J)
   RATIO(J)=M(J)/MPC
33 Z(I)=Z(I)+RHO
   IF(RATIO(J) .GE. 1.0) GO TO 30
   IF(RHO .EQ. 1.0 .AND. A(J) .GE. 0.0000) GO TO 36
   IF(A(J) .LT. 0.0000) GO TO 30
36 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AO,RPHI(J)
   J=J+1
   IF(RHO .EQ. 1.0) GO TO 32
   GO TO 34
30 WRITE(6,2) I,J,V(J),RHO,Z(I),RMM(J),RATIO(J),A(J),AO,RPHI(J)
31 CONTINUE
   RETURN
   END

```

TABLE 9

Results for $P = 0.6P_y$ and $\alpha_0 = 0.065$

I	J	V(J)	RHO(J)	Z(J)	RMM(J)	MJ/MPC	A(J)	A(0)	RPHI(J)
7	1	0.51852	8.00000	8.00000	0.15464	0.31996	0.06463	0.06500	0.15464
7	2	1.07832	8.00000	16.00000	0.46216	0.63453	0.06282	0.06500	0.75302
7	3	1.50513	8.00000	24.00000	0.76109	0.92876	0.05638	0.06500	2.68294
7	4	1.55637	1.00000	25.00000	0.91200	0.96038	0.04610	0.06500	34.27789
7	5	1.59552	1.00000	26.00000	0.93941	0.98454	0.03220	0.06500	46.34225
7	6	1.61950	1.00000	27.00000	0.95856	0.99933	0.01577	0.06500	54.76822
7	7	1.62644	1.00000	28.00000	0.96794	1.00361	-0.00190	0.06500	58.89426

DEFLECTION CURVES FOR COLUMNS

BENDING ABOUT THE WEAK AXIS

ABSTRACT

by Sampath N.S. Iyengar, M.S.
Washington State University, 1966

Chairman: Loren B. Almy

Deflection Curves for columns bending about the weak axis under external end moments (symmetrical single curvature) and in the presence of axial thrusts (1) $P = 0.2 P_y$ and (2) $P = 0.6 P_y$ have been developed using the concept of Column Deflection Curves or CDC's. Strengths of columns with slenderness ratios 20, 30, and 40 have been estimated and the results compared with those for bending about the strong axis.