

ANALYSIS OF COMPOSITE ACTION IN LUMBER DIAPHRAGMS

By

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A thesis submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

WASHINGTON STATE UNIVERSITY
College of Engineering

1979

ACKNOWLEDGMENTS

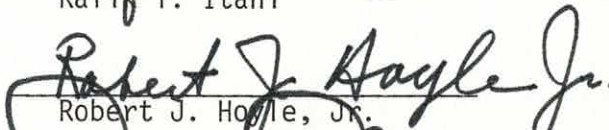
The author is in gratitude to Dr. R. Y. Itani for his constant encouragement and guidance throughout the course of this study. He is also grateful to Professor R. J. Hoyle for his valuable guidance and interest in this work. Sincere appreciation is extended to Professor J. F. Orsborn and Professor L. D. Luck for their encouragement.


The author wishes to acknowledge deeply his appreciation to Mr. S. Hall for his valuable suggestions in preparing the manuscript and for proofreading the manuscript. Mrs. Sandy Jack's meticulous job in typing the manuscript.

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ACKNOWLEDGMENTS

The author is in gratitude to Dr. R. Y. Itani for his constant encouragement and guidance throughout the course of this study. He is also grateful to Professor R. J. Hoyle for his valuable guidance and interest in this work. Sincere appreciation is extended to Professor J. F. Orsborn and Professor L. D. Luck for their encouragement.

The author wishes to acknowledge deeply his appreciation to Mr. B. Naik for his valuable suggestions in preparing the manuscript and for proofreading the manuscript. Mrs. Sandy Tyacke's meticulous job in typing the manuscript is also sincerely appreciated.

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ABSTRACT

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Washington State University, 1979

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Analysis of composite action in lumber diaphragms is presented herein. The diaphragms are analyzed as a multilayer system bonded with elastomeric adhesives. A computer program employing finite difference methods to solve the governing set of differential equations is developed and includes the ability to compute strains and deflections of the system.

It is concluded that in a diaphragm all the adjacent layers need not be bonded with shear connectors.

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CHAPTER 1

INTRODUCTION

Diaphragms are common and economical structural systems often used to resist lateral forces acting on a structure. These forces may be due to wind loads, earthquake and blasts.

A diaphragm is a large flat structure acting in the response to lateral loads. Structural components act as diaphragms when they resist loads in their own plane. Common examples of diaphragms are roof decks or floor system, resisting loads in their own planes or folded plate roofs when they resist lateral loads.

In practice, a diaphragm is designed as a composite structure consisting of several courses. The number of courses acting together which form a diaphragm are connected by means of a 'shear connector system.' The shear connector system may be of any kind, such as nails or adhesives, in case of wood diaphragms; or the different metallic shear connectors which are used in pre-cast construction. Under the action of load, the shear connector system often fails to impart a rigid bond to the adjacent courses so that the diaphragm does not act as a 'monolithic' system. The shear connector system undergoes deformation resulting in 'Interlayer Slip' between the adjacent courses.

Any structural analysis of a diaphragm must account for the deformation in the shear connector system. As a result, the structural analysis of a diaphragm, which is a highly indeterminate system, poses a complex problem.

Consequently, design guideline diaphragms are mainly based on experience and testings.

The aim of this study is to analyze the behavior of diaphragms with varying parameters, such as the total number of courses and the stiffness of the shear connector system. Though the analysis is restricted to wooden diaphragms with elastomeric adhesives as the shear connector system, it can be extended to diaphragms of any material having parallel courses bonded by any kind of shear connector system. This study does not account for openings in the diaphragms.

The development of the governing set of differential equations is the soul of the study. The basis of the above development is the pioneering work by Newmark, Seiss and Viest [17] and the subsequent study by Hoyle and McGee [10] on a two-layer beam bonded with a continuous glue line. A brief review of the major earlier work is given in Chapter 2, proceeded by a discussion of the original formulation of the governing differential equation for a two-layer system in Chapter 3. The development of the governing set of differential equations for a multi-layer diaphragm is presented in Chapter 4.

Finite difference technique is employed to solve the above mentioned governing set of differential equations and is discussed in Chapter 5. A computer program DAD--Difference Analysis of Diaphragms written in FORTRAN is developed to solve the governing set of differential equations and for related studies. Appendix A presents the DAD user's guide with examples in Appendix B.

The results of the theoretical analysis of diaphragms are compared with the results of the experimental work done by Johnson [14] and discussed in Chapter 6. Further discussions of results of the present study followed by conclusions and remarks are presented in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

A brief review of the major previous development leading to the study of diaphragms is presented in this chapter. The study of diaphragms with layers can be broadly classified in the area of partial composite action or incomplete interaction between structural components. The literature review is related to this area too.

Although substantial work has been done in the area of partial composite action, an organized bibliography was published only recently by Carney [2] on wood and plywood diaphragms. This bibliography shows the dominance of experimental analyses in the study of diaphragms. In this connection, the work by Johnson [13, 14] is worth mentioning.

The basis of the theoretical study of diaphragms could be traced back to the pioneering work of Newmark et al. [17] in 1951, who studied the incomplete interaction of a composite T beam having a concrete deck connected to a steel I-beam by shear connectors. They formulated a second order differential equation to express the relationship between the interlayer slip and the longitudinal force in each element of the T-beam. A few years later, Clark [3] reported the study on laminated beams where he employed moment-area method to compute the deflection of a beam with two or more laminations. Goodman [6] studied extensively the interlayer slip problem involving layered systems.

Hoyle and McGee [10] developed a closed-form solution of the second-order differential equation relating the interlayer slip and the longitudinal

force in each layer of a two-layer beam bonded with a continuous glueline. They considered the case of a lateral concentrated load acting on a simply supported beam. Similar solutions were also reported by Venderbilt [18]. Extension of the above approach to a three-layer system subjected to a uniformly distributed load was done by Anderson [1]. Itani and Brito [12] developed a closed form solution for a two-layer system with gaps. The inter-layer slip in the above study is applicable to both adhesive bonds and mechanical connector links. The only difference is that in the case of adhesives, the width of the glueline must be accounted for. Otherwise, the concept of 'slip-modulus,' which is the force per unit slip, was important. Some of the assumptions made in the above studies are common, which are as follows:

1. Deflections are small.
2. Separation between the adjacent layers does not occur.
3. Friction between the adjacent layers is negligible.
4. Strain distribution is linear over the depth of the layer.
5. Slip is directly proportional to the load transmitted to the connectors.
6. The connectors holding together any two layers along the length of the beam have same capacity.
7. Slip modulus is constant at all points between neighboring layers.
8. The layers have constant properties all over the length.
9. Shear connection between the layers is assumed to be continuous along the length of the beam.
10. The layers have an equal curvature before the loading.

Besides closed form solutions mentioned above, numerical methods employing techniques such as finite difference and finite elements have been used. Ko [15] applied finite difference technique to solve the governing equations for partial composite beams having single axis symmetry and arbitrary number

of layers fastened together with mechanical connectors. In the same study, he analyzed two and three layer systems in detail for the effect of interlayer connections on deflections. Foschi [5] developed a computer program to analyze the diaphragms based on finite element techniques. His study accounted for the orthotropic plate action and nonlinear connection behavior. The analysis gives good estimates for deformation of diaphragms and ultimate load related to connection yielding.

Along with the studies mentioned above, there has been considerable progress in the research on the shear connector system. Although the use of adhesive as a shear connector system has been known since 1940, it is being used extensively recently. This is because adhesives impart a bond several times more rigid than that imparted by nails. So researchers now focus their attention on the properties and structural behavior of not only nails, but also adhesives, particularly elastomeric adhesives. An important aspect is the study of interlayer slip wherein relationships between the loads and the resulting deformations of the shear connector system are established. The works of several investigators in this area are worth mentioning. Foschi [4] studied the deformation and the load relationship to be nonlinear in the light of nail penetration and load direction. Hoyle et al. have studied the properties and the behavior of elastomeric adhesives extensively. Hoyle and Dong [8] studied the utility of the different glues regarding the shear modulus, shear strength and recovery characteristics at 70°F temperature. The study indicated that static tests for shear modulus and shear strength are not sufficient. Hoyle and Hsu [9] concluded that the cyclic loading decreases the shear strength and shear modulus, but most of the losses in the shear strength take place in the first ten loadings, which amount to about seven percent, while shear modulus reduces to between ten and fifteen percent during the first

seven cycles of stress. This is an important conclusion in the light of utility of elastomeric adhesives in wood structures. Kuenzi and Wilkinson [16] suggested practical ways of measuring the shear modulus. Hsu and Hoyle [11] studied the factors affecting the measurement of shear modulus of an elastomeric adhesive. They suggested that the design value of shear modulus of a glue could be determined by some cyclic loading. Hoyle [7] in his discussion on the design of wood structures bonded with elastomeric adhesives recommended shear strength of 50 psi to 200 psi for home building application.

The following assumptions are made in the analysis of the system:

1. The adhesive is perfectly elastic and its modulus is constant throughout the length of the system.
2. The adhesive is perfectly bonded to the wood and there is no slip at the interface.
3. The adhesive is perfectly bonded to the wood and there is no slip at the interface.

The shear stress τ in the adhesive is directly proportional to the shear strain γ and is given by the following equation:

$$\tau = G \gamma$$

where G is the shear modulus of the adhesive and γ is the shear strain.

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CHAPTER 3

DEVELOPMENT OF THE GOVERNING EQUATION FOR A TWO-LAYER MODEL

The development of the governing equation for a two-layer model (Fig. 3.1) discussed herein, is for any general type of lateral in-plane loading condition. The two layers are glued together by an adhesive of finite rigidity. Though the discussion and the equation developed pertain to a glue bonded system, the same can be applied to a mechanical shear connector bonded system. The equivalence between the stiffness of a glue and the capacity of a mechanical connector system is briefed at the later part of the chapter.

3.1. Assumptions Made for the Analysis of a Two-Layer Model

The analysis is based upon the following assumptions:

1. The bond between the layers is continuous and uniform along the length of the beam.
2. The amount of slip in the adhesive is directly proportional to the load transmitted.
3. The strain distribution is linear across the depth of the layer.
4. Each layer is continuous throughout the length of the system.
5. The layers have equal curvature before the loading and after the loading.
6. Stress-Strain relationship is linear.

3.2. Analysis of the Two-Layer Model

If the two layers in the system are unbonded, each layer will behave independently. The stress in the extreme fiber of each of the layers will be

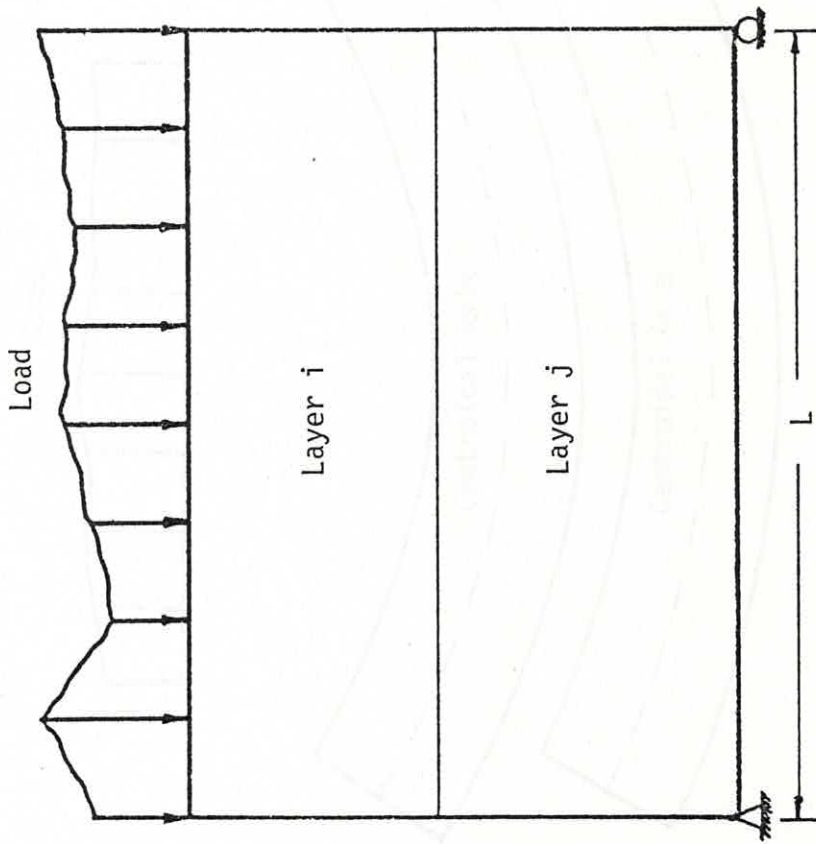


Fig. 3.1.--Two-layer model.

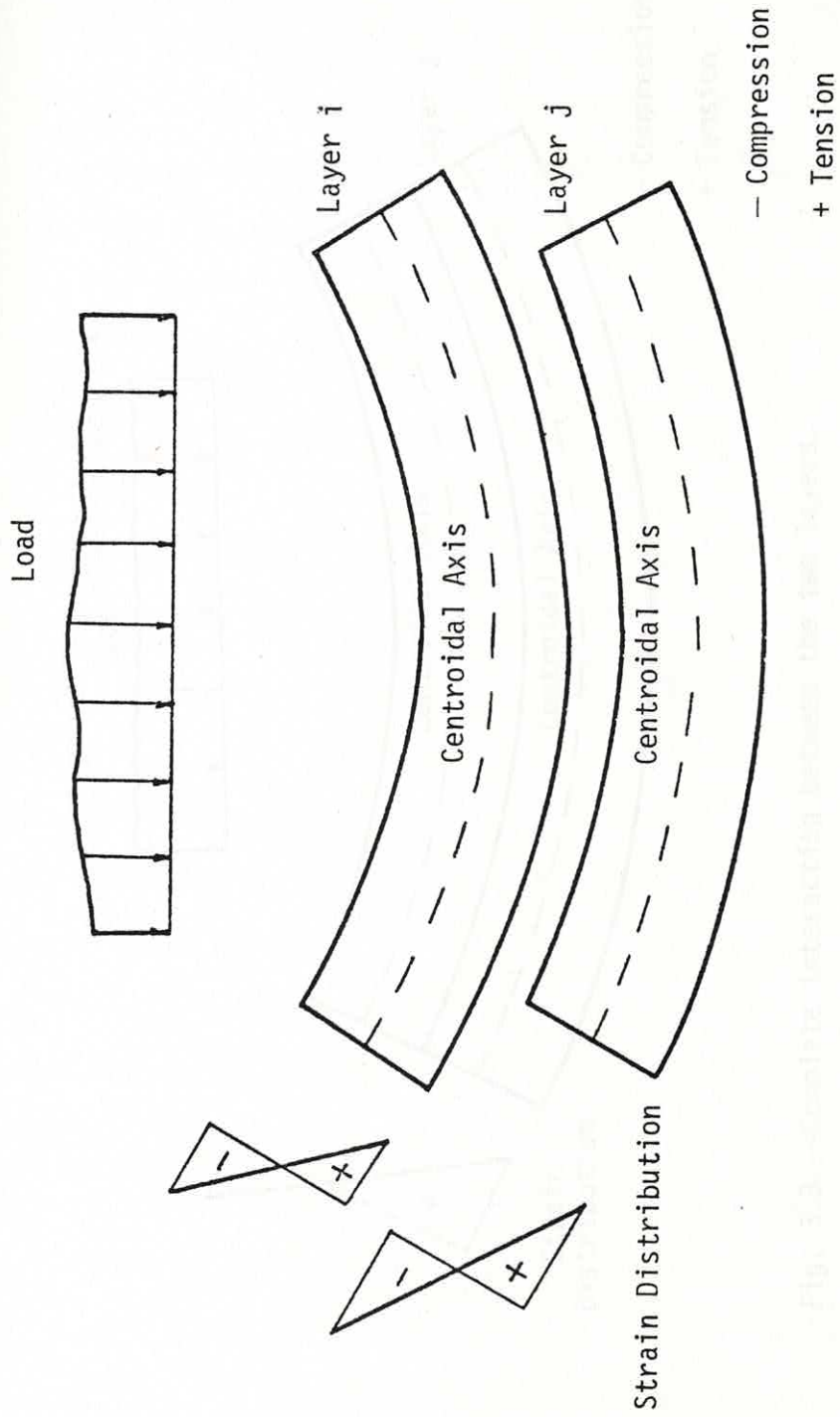


Fig. 3.2.--No interaction between the layers.

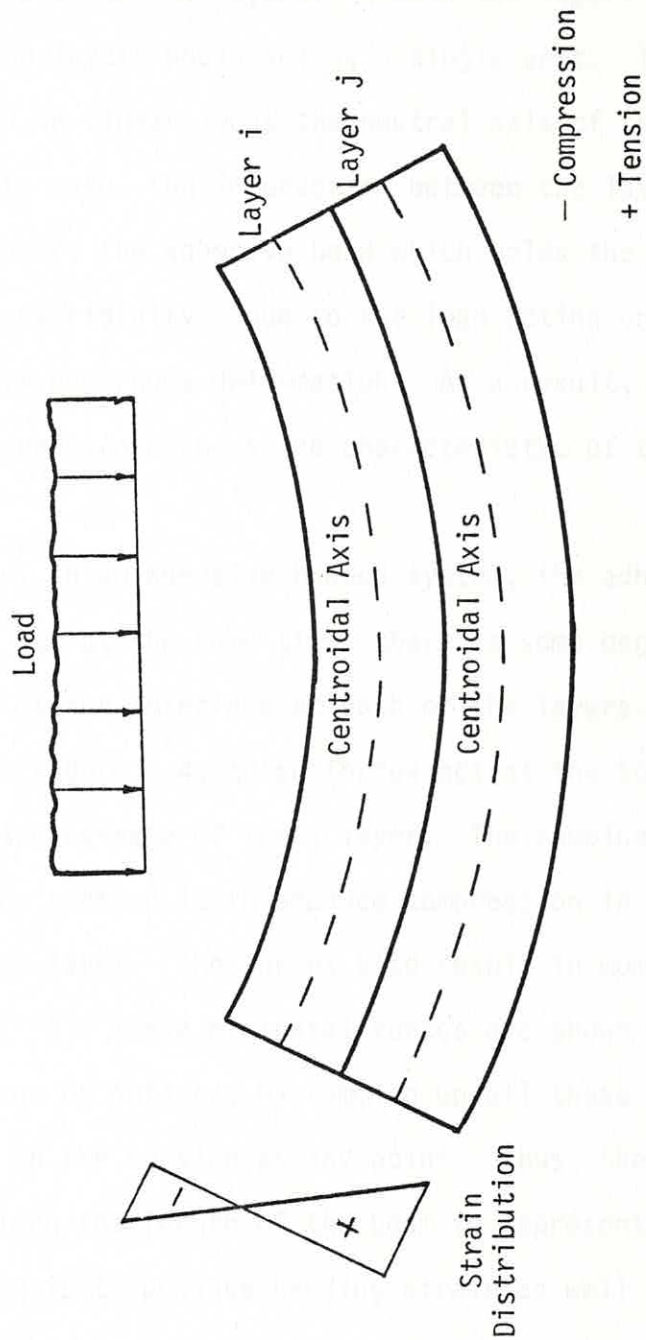


Fig. 3.3.--Complete interaction between the two layers.

equal in magnitude and opposite in nature (Fig. 3.2). The stress distribution is linear across the depth of each layer. In this case, there is no interaction between the layers. Should the layers be held together by a rigid bond, the two layers would act as a single unit. In that case, strain distribution would be linear about the neutral axis of the 'single unit,' (Fig. 3.3). Thus, in this case, the interaction between the layers is said to be complete.

However, the adhesive bond which holds the layers together does not impart perfect rigidity. Due to the load acting on the beam, the shear connector system undergoes deformation. As a result, an intermediate case of partial interaction becomes the characteristic of this two-layer model (Fig. 3.4).

Thus, in an adhesive bonded system, the adhesives do not impart a rigid bond, but at the same time, there is some degree of interaction at the interface. At the interface at each of the layers, shear forces are developed. Referring to Figure 3.4, these forces act at the bottom surface of the i layer and at the top surface of the j layer. The combined effect of the shear forces at the section is to produce compression in the top layer and tension in the bottom layer. The forces also result in moments about the centroid of the section. All these elemental forces are shown by dF in Figure 3.4. The net result can be obtained by summing up all these elemental forces to obtain total force on the section at any point. Thus, the elemental forces can be summed up along the length of the beam to represent a total force. The effect of this force is to produce bending strain as well as axial strain on each layer. So, this force can be represented by a couple acting at the centroid of each layer (Fig. 3.6). Then, net moment at any section along the length of the beam is given by,

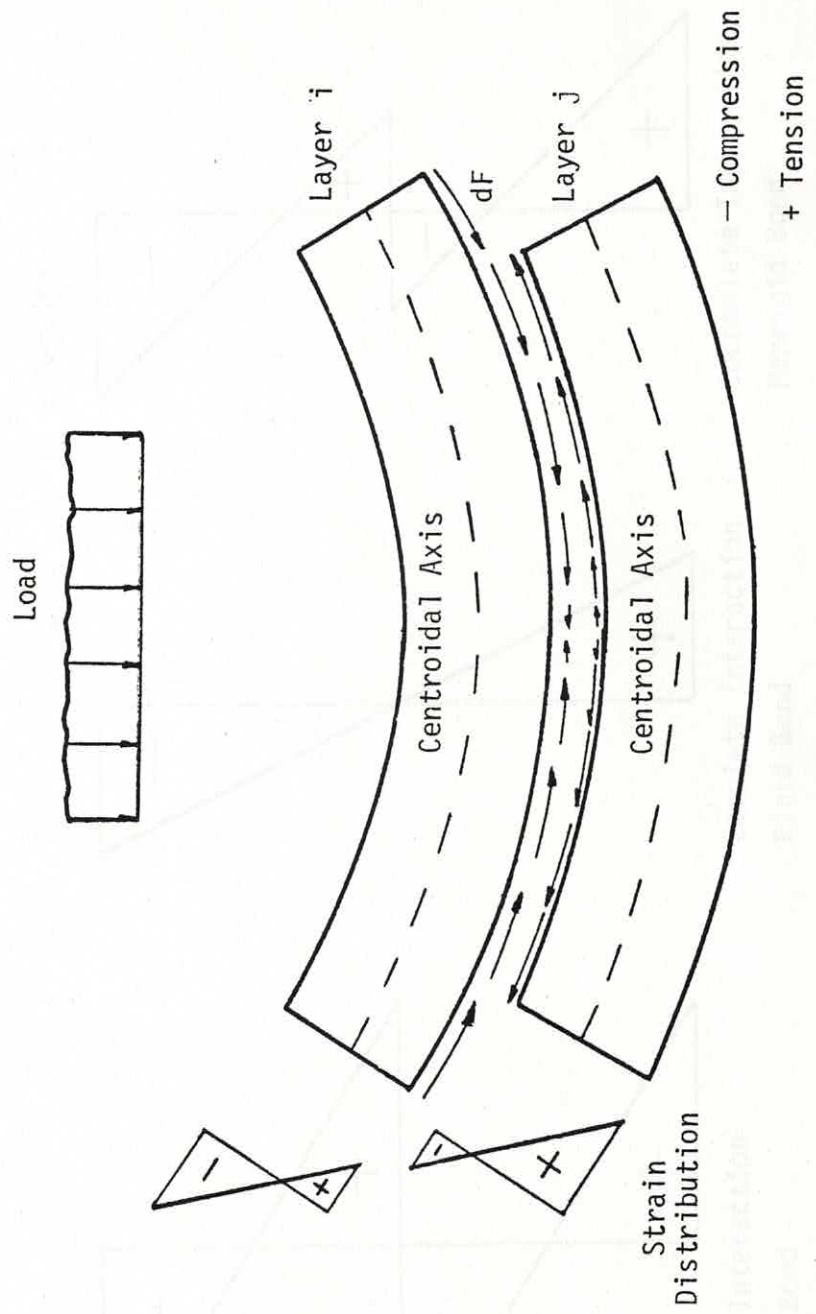


Fig. 3.4.--Partial interaction between the layers.

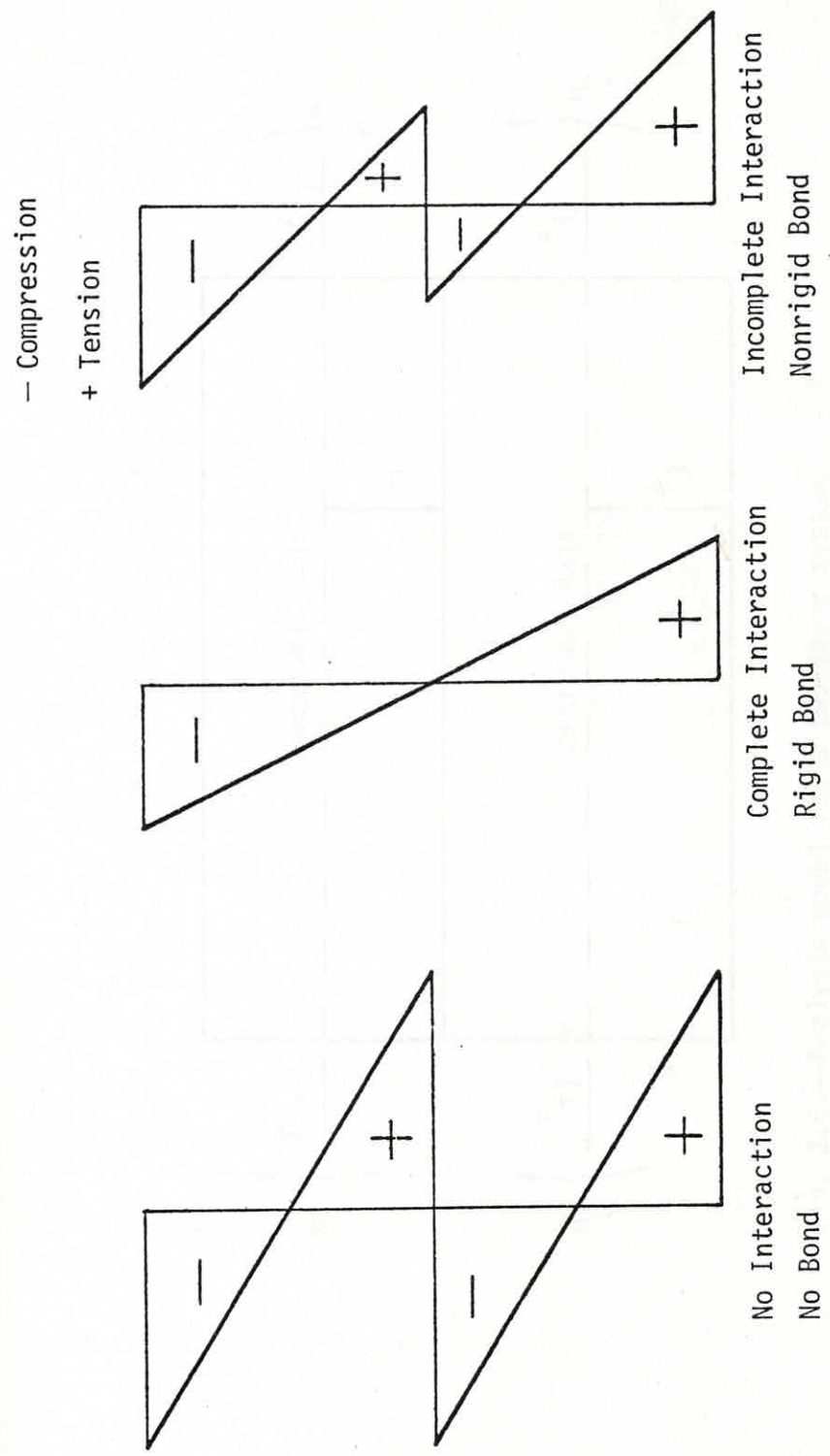


Fig. 3.5.--Strain distribution in two-layer system for different bond rigidity.

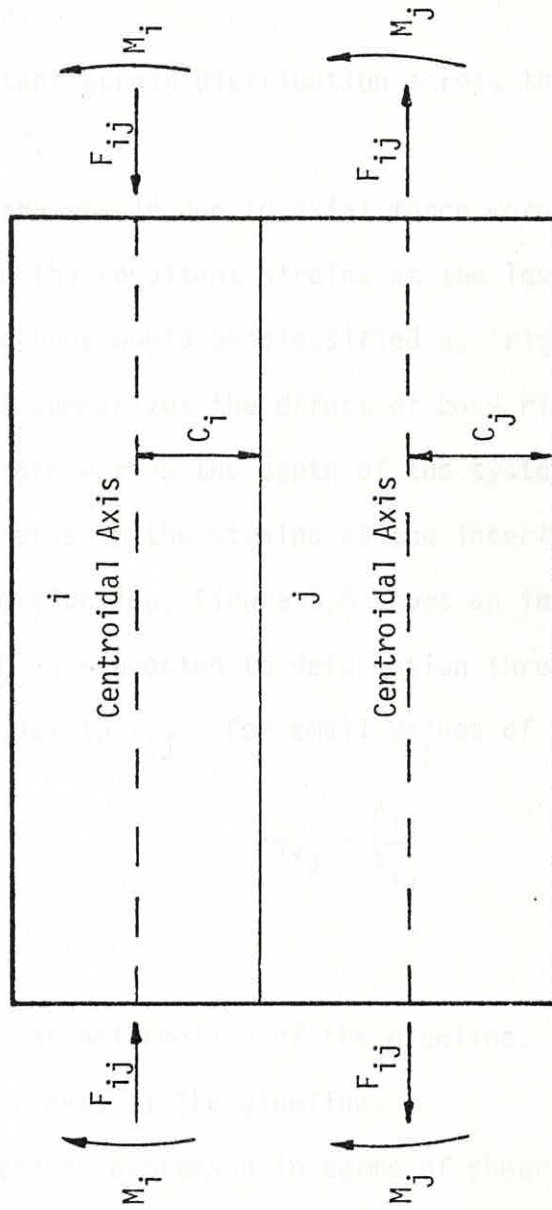


Fig. 3.6.--Analysis model for a two-layer system.

$$M_i + M_j = M_x - F_{ij}(C_i + C_j) \quad (3.1)$$

where, M_x is the moment at that section caused by the external load; C_i and C_j are the respective distances of the extreme fibers from centroidal axis of the i and j layers.

The resultant strain distribution across the depth of the beam is shown in Figure 3.7.

Thus, if the strain due to axial force were large enough to reduce the difference between the resultant strains at the level of interface of the two layers to zero, the bond would be classified as 'rigid bond,' (Fig. 3.5).

Figure 3.5 summarizes the effect of bond rigidity of different degrees on the strain pattern across the depth of the system.

The difference in the strains at the interface level is caused by the deformation of the glueline. Figure 3.8 shows an infinitesimal length of the glueline dx , which is subjected to deformation through angle γ_{ij} . Thus, the shear strain is equal to γ_{ij} . For small values of γ_{ij} ,

$$\gamma_{ij} = \frac{\Delta_{ij}}{t_{ij}} \quad (3.2)$$

where,

Δ_{ij} is the linear deformation of the glueline.

t_{ij} is the thickness of the glueline.

The shear strain can be expressed in terms of shear stress as

$$\gamma_{ij} = \frac{\tau_{ij}}{G_{ij}} \quad (3.3)$$

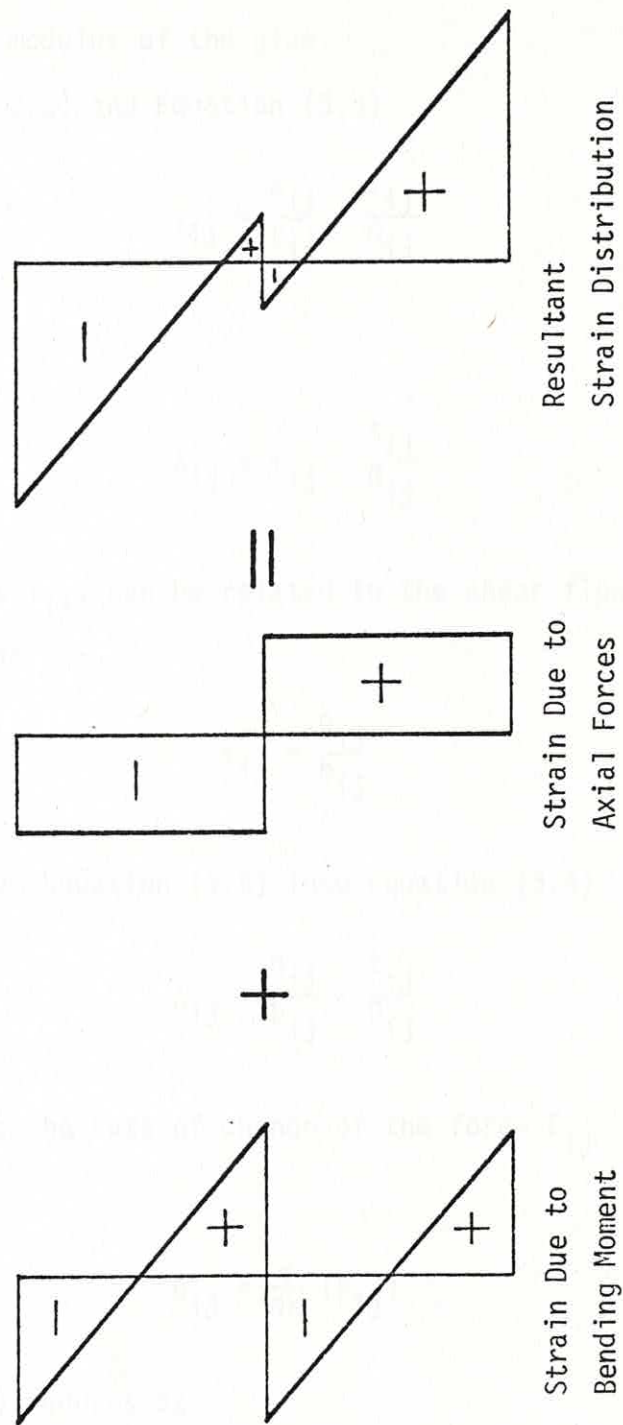


Fig. 3.7.--Strain distributions in two-layer model.

where,

τ_{ij} is the shear stress in the glueline.

G_{ij} is the shear modulus of the glue.

Thus, from Equation (3.2) and Equation (3.3)

$$\gamma_{ij} = \frac{\Delta_{ij}}{t_{ij}} = \frac{\tau_{ij}}{G_{ij}}$$

Rearranging,

$$\Delta_{ij} = \tau_{ij} \frac{t_{ij}}{G_{ij}} \quad (3.4)$$

But, the shear stress τ_{ij} , can be related to the shear flow q_{ij} and the width of the glueline b_{ij} as

$$\tau_{ij} = \frac{q_{ij}}{b_{ij}} \quad (3.5)$$

Substituting, τ_{ij} from Equation (3.5) into Equation (3.4)

$$\Delta_{ij} = \frac{q_{ij}}{b_{ij}} \cdot \frac{t_{ij}}{G_{ij}} \quad (3.6)$$

The shear flow q_{ij} is the rate of change of the force F_{ij} along the length of the beam, i.e.,

$$q_{ij} = \frac{d}{dx} (F_{ij}) \quad (3.7)$$

Hence, Equation (3.6) reduces as

$$\Delta_{ij} = \frac{d}{dx} (F_{ij}) \cdot \frac{1}{\left(\frac{G_{ij} b_{ij}}{t_{ij}} \right)} \quad (3.8)$$

The term $\left(\frac{G_{ij} b_{ij}}{t_{ij}} \right)$ is the stiffness of the glue line and henceforth is represented by s_{ij} . Thus, Equation (3.8) is rewritten as

$$\Delta_{ij} = \frac{1}{s_{ij}} \cdot \frac{d}{dx} (F_{ij}) \quad (3.9)$$

The stress-strain relationship is assumed to be linear, therefore,

$$\text{Strain} = \frac{\text{Stress}}{\text{Modulus of Elasticity}}$$

Here the stress is a combination of the bending and axial stresses. The rate of change of slip along the length of the beam can be obtained by differentiating Δ_{ij} in the Equation (3.9) with respect to x

$$\frac{d\Delta_{ij}}{dx} = \frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} \quad (3.10)$$

But, the rate of change of slip is the residual strain at the level of the interface. Mathematically,

$$\frac{d\Delta_{ij}}{dx} = \epsilon_{j_t} - \epsilon_{i_b}$$

where,

ϵ_{j_t} is the strain in the top fibers of the layer j

ϵ_{i_b} is the strain in the bottom fibers of the layer i

Hence, Equation (3.10) reduces as

$$\frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} = \epsilon_{j_t} - \epsilon_{i_b} \quad (3.11)$$

Equation (3.11) relates the strains in the extreme fibers of the two layers in contact and the force that would develop out of interaction at the interface.

It can be observed that if the strains in two layers at the interface level were equal in magnitude, the stiffness of the glueline must be infinitely large. Such a case is hypothetically possible if the glueline has an infinite modulus of rigidity or the thickness of the glueline is zero. This implies that a glueline cannot provide a rigid bond.

Equation (3.11) is identical to that developed by Newmark et al. [17] in the study of partial interaction between a precast slab and a steel beam. The further development is similar to the governing equation developed by Hoyle and McGee [10] for a two-layer adhesive bonded system.

Equation (3.11) can be developed to a convenient form by substituting the e_{j_t} and e_{i_b} in terms of stresses.

$$e_{j_t} = \frac{F_{ij}}{E_j A_j} - \frac{M_j C_j}{E_j I_j}$$

$$e_{i_b} = \frac{-F_{ij}}{E_i I_i} + \frac{M_i C_i}{E_i I_i}$$

where,

F_{ij} is the axial force acting at the centroid of each layer.

M_i, M_j are the net moments acting on the layer i and the layer j , respectively.

C_i, C_j are the centroidal distances from the extreme fibers of the layer i and the layer j , respectively.

E_i, E_j are the modulus of elasticity of the layer i and the layer j , respectively.

I_i, I_j are the moment of inertia of the cross section of the layer i and the layer j about their own centroidal axis, respectively.

Substituting in Equation (3.11)

$$\frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} = \left(\frac{F_{ij}}{E_j A_j} - \frac{M_j C_j}{E_j I_j} \right) - \left(\frac{-F_{ij}}{E_i I_i} + \frac{M_i C_i}{E_i I_i} \right)$$

Rearranging,

$$\frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} = F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} \right) - \left(\frac{M_j C_j}{E_j I_j} + \frac{M_i C_i}{E_i I_i} \right) \quad (3.12)$$

Since, the layers have the same curvature, the following relationship can be concluded.

$$\frac{M_i}{E_i I_i} = \frac{M_j}{E_j I_j} \quad (3.13)$$

But, Equation (3.1) gives

$$M_i + M_j = M_x - F_{ij}(C_i + C_j)$$

Then Equation (3.13) can be extended as

$$\frac{M_i}{E_i I_i} = \frac{M_j}{E_j I_j} = \frac{M}{EI} = \frac{M_x - F_{ij}(C_i + C_j)}{E_i I_i + E_j I_j} \quad (3.14)$$

where,

M , M_x are the net moment and the moment due to the external load at any section along the length of the system, respectively.

$$M = M_i + M_j$$

$$M_i + M_j = M_x - F_{ij}(C_i + C_j)$$

EI is the sum of the products of the moment of inertia and the modulus of elasticity of each layer, i.e.,

$$EI = E_i I_i + E_j I_j$$

From the above relationship,

$$\frac{M_j C_j}{E_j I_j} = \frac{C_j [M_x - F_{ij} (C_i + C_j)]}{E_i I_i + E_j I_j}$$

$$\frac{M_i C_i}{E_i I_i} = \frac{C_i [M_x - F_{ij} (C_i + C_j)]}{E_i I_i + E_j I_j} \quad (3.14)$$

Substituting Equation (3.14) in Equation (3.12)

$$\frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} = F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} \right) - \frac{(C_i + C_j) [M_x - F_{ij} (C_i + C_j)]}{E_i I_i + E_j I_j}$$

Collecting terms and rearranging,

$$\frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} = F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} + \frac{(C_i + C_j)(C_i + C_j)}{E_i I_i + E_j I_j} - \frac{M_x (C_i + C_j)}{E_i I_i + E_j I_j} \right) \quad (3.15)$$

Equation (3.15) is the governing equation for a two-layer system.

Equation (3.15) is a second order linear differential equation which can be solved for the force F_{ij} , at a particular section along the length of the beam from the known value of bending moment at that section. Once, the force F_{ij} is known, going back to Equations 3.5 to 3.8, the required values of the stresses, strains, etc. can be calculated. The deflection can be obtained by the fundamental equation,

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

which yields to when y is the deflection of the section.

$$\frac{d^2 y}{dx^2} = \frac{M_x - F_{ij}(C_i + C_j)}{E_i I_i + E_j I_j} \quad (3.16)$$

The closed form solution for the Equation (3.15) is developed for the following cases:

1. Simply supported system with a concentrated lateral in-plane load, Hoyle and McGee [10].
2. Simply supported system with a uniformly distributed lateral in-plane load, Anderson Mark [1].

In Equation (3.15), the term s_{ij} stands for the stiffness of the glue-line. By the definition

$$s_{ij} = \frac{G_{ij} \cdot b_{ij}}{t_{ij}}$$

where,

G_{ij} is the shear modulus of the glue.

b_{ij} and t_{ij} , respectively, are the width and the thickness of the glue-line.

If the glue were to be replaced by a mechanical connector system, the term s_{ij} would be replaced by its equivalent for the mechanical connector system. The stiffness of the connectors holding together the layer i and the layer j , say ρ_{ij} is given by

$$\rho_{ij} = \frac{k_{ij} \cdot n_{ij}}{d_{ij}}$$

where,

k_{ij} is the slip modulus of connector.

n_{ij} is the number of connectors per row.

d_{ij} is the spacing of connectors.

Thus, for a glued system, the area on which the glue acts is accounted for, while that is not the case with the connectors. The dimensions of s_{ij} and ρ_{ij} are $[F^1 L^{-2} T^0]$. The dimensions of the associated quantities are given in Table 3.1.

TABLE 3.1.--Dimensions of the quantities associated with the stiffness of a shear connector system.

Quantity	Dimensions
G_{ij}	$F^1 L^{-2} T^0$
b_{ij}	$F^0 L^1 T^0$
t_{ij}	$F^0 L^1 T^0$
k_{ij}	$F^1 L^{-1} T^0$
d_{ij}	$F^0 L^1 T^0$
n_{ij}	$F^0 L^0 T^0$

Thus, the second order differential equation developed is applicable to any two-layer system with a nonrigid bond, irrespective of the type of shear connector system.

CHAPTER 4

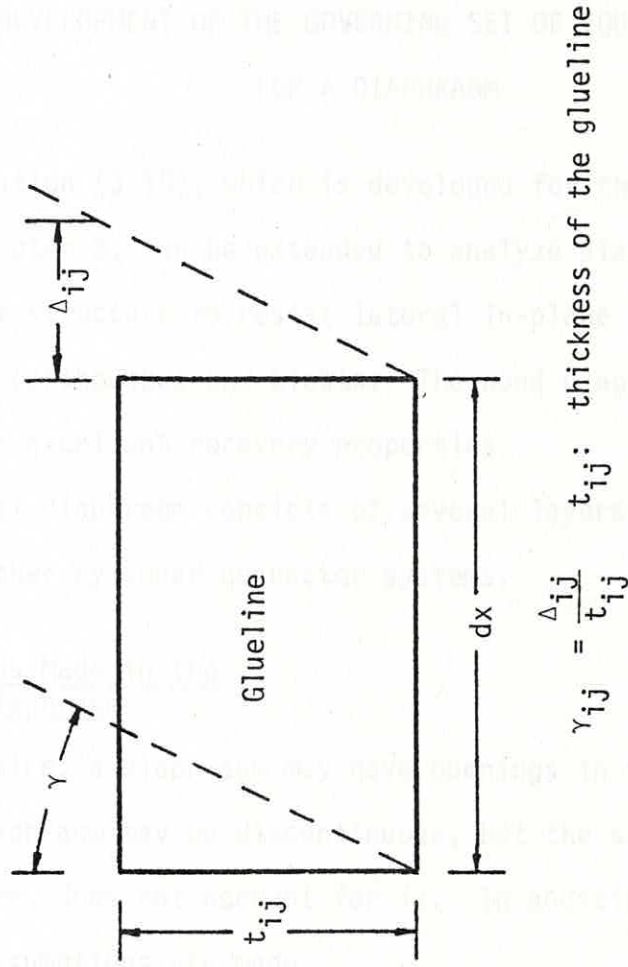


Fig. 3.8.--Glue line deformation.

1. There are no openings in the diaphragm.
2. There are no transverse members.
3. The layers are continuous in the length of the diaphragm.
4. Inertia of plates is neglected.
5. The only load acting on the diaphragm is the lateral load in the plane of a diaphragm.
6. Shrinkage effects, etc. are neglected.

CHAPTER 4

DEVELOPMENT OF THE GOVERNING SET OF EQUATIONS

FOR A DIAPHRAGM

The Equation (3.15), which is developed for the two-layer model as discussed in Chapter 3, can be extended to analyze diaphragms. A diaphragm is a large, flat structure to resist lateral in-plane loads. The loads may be due to wind, earthquakes and blasts. The wood diaphragms are popular because of their excellent recovery properties.

A typical diaphragm consists of several layers of beam elements which are bonded together by shear connector systems.

4.1. Assumptions Made in the Analysis of a Diaphragm

In practice, a diaphragm may have openings in it and also the layers which form a diaphragm may be discontinuous, but the simplified approach, that is presented here, does not account for it. In addition to those in Chapter 3, the following assumptions are made:

1. There are no openings in the diaphragm.
2. There are no boundary members.
3. The layers are continuous all over the length of the diaphragm.
4. Effect of joists is neglected.
5. The only load acting on the diaphragm is the lateral load in the plane of a diaphragm.
6. Shrinkage effect, etc. are neglected.

Thus, these assumptions reduce the diaphragm to a simple multilayer system bonded with shear connector system. In case of a wood diaphragm, the different layers are glued together. Thus, a number of two-layer models discussed in Chapter 3 acting together form a wood diaphragm. As a result, a set of governing equations is obtained which predicts the behavior of diaphragms.

4.2. Analysis of an Equivalent Multilayer System

The diaphragm resists a part of the wind, or earthquake, or forces out of blasts. Figure 4.1 is a simplified model of a diaphragm. The different layers g, h, i, j, \dots, m are glued together by adhesives. This system can be looked upon as an assembly where layer i , and layer j of the two-layer model are sandwiched by a set of layers at the top and a set of layers at the bottom. Each glueline in this system undergoes deformation as the load acts on the diaphragm. As a result, at each of the interface, there exists a non-rigid bond which will impart partial interaction between the two layers in contact with each other. The shear force, resulting out of the partial interaction between any two layers which are in contact with each other, can be replaced by a couple acting at the centroid of the two layers, as in case of the two-layer model in Figure 3.6. The effect of the external load and the inter-layers slip can be replaced by an equivalent system of couples and moments, which will account for the deformation of the gluelines in the system shown in Figure 3.2. Thus, the net moment acting at any section along the length of the diaphragm, is the sum of the moment resisted by each layer. The net moment can be related to the external moment as follows:

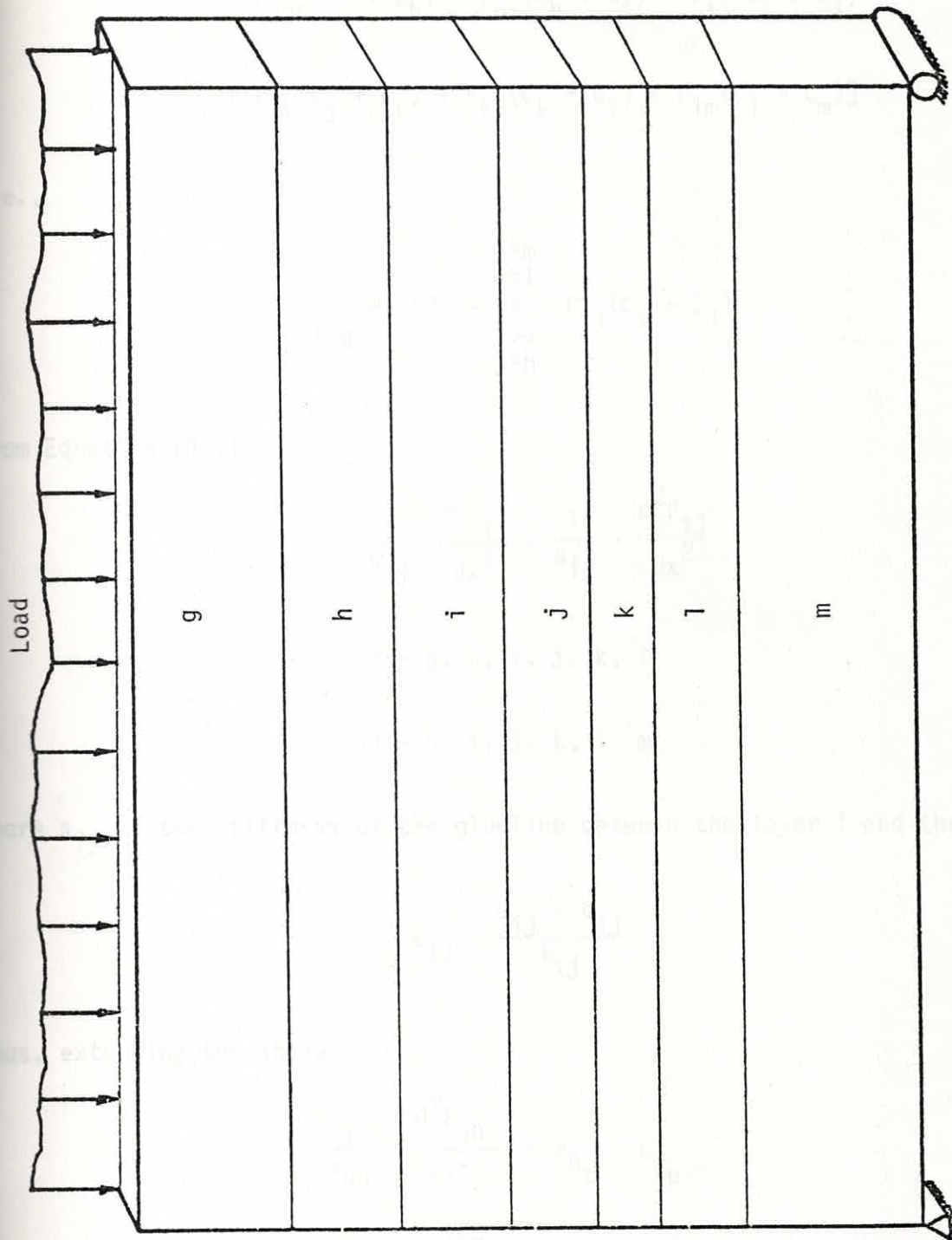


Fig. 4.1.--Multilayer system: diaphragm model.

$$\begin{aligned}
 & M_g + M_n + M_i + M_j + M_k + M_l + M_m = M_x \\
 & - [F_{gh}(C_g + C_h) + F_{hi}(C_h + C_i) + F_{ij}(C_i + C_j) \\
 & + F_{jk}(C_j + C_k) + F_{kl}(C_k + C_l) + F_{lm}(C_l + C_m)]
 \end{aligned}$$

i.e.,

$$\sum_{i=g}^m M_i = M_x - \sum_{\substack{j=m \\ i=l \\ i=g \\ j=h}} F_{ij}(C_i + C_j) \quad (4.1)$$

from Equation (3.11)

$$\frac{1}{s_{ij}} \frac{d^2 F_{ij}}{dx^2} = \frac{1}{s_{ij}} \cdot \frac{d^2 F_{ij}}{dx^2}$$

$$i = g, h, i, j, k, l$$

$$j = h, i, j, k, l, m$$

where s_{ij} is the stiffness of the glue line between the layer i and the layer j

$$s_{ij} = \frac{G_{ij} \cdot b_{ij}}{t_{ij}}$$

Thus, extending the above

$$\frac{1}{s_{gh}} \cdot \frac{d^2 F_{gh}}{dx^2} = \epsilon_{ht} - \epsilon_{gb} \quad (4.2.1)$$

$$\frac{1}{s_{hi}} \cdot \frac{d^2 F_{hi}}{dx^2} = \epsilon_{it} - \epsilon_{hb} \quad (4.2.2)$$

$$\frac{1}{s_{ij}} \cdot \frac{d^2 F_{ij}}{dx^2} = \epsilon_{jt} - \epsilon_{ib} \quad (4.2.3)$$

$$\frac{1}{s_{jk}} \cdot \frac{d^2 F_{jk}}{dx^2} = \epsilon_{kt} - \epsilon_{jb} \quad (4.2.4)$$

$$\frac{1}{s_{kl}} \cdot \frac{d^2 F_{kl}}{dx^2} = \epsilon_{lt} - \epsilon_{kb} \quad (4.2.5)$$

$$\frac{1}{s_{lm}} \cdot \frac{d^2 F_{lm}}{dx^2} = \epsilon_{mt} - \epsilon_{lb} \quad (4.2.6)$$

Substituting the values of the strains in the above set of Equations (4.2) in terms of forces and moments and rearranging the R.H.S. in Equations (4.2) can be simplified as follows: (referring to Fig. 4.2)

$$\frac{1}{s_{gh}} \cdot \frac{d^2 F_{gh}}{dx^2} = 0 + F_{gh} \left(\frac{1}{E_g A_g} + \frac{1}{E_h A_h} \right) + F_{hi} \left(\frac{-1}{E_h A_h} \right) - \left(\frac{M_g C_g}{E_g I_g} + \frac{M_h C_h}{E_h I_h} \right) \quad (4.3.1)$$

$$\frac{1}{s_{hi}} \cdot \frac{d^2 F_{hi}}{dx^2} = F_{gh} \left(\frac{-1}{E_h A_h} \right) + F_{hi} \left(\frac{1}{E_h A_h} + \frac{1}{E_i A_i} \right) + F_{ij} \left(\frac{-1}{E_i A_i} \right) - \left(\frac{M_h C_h}{E_h I_h} + \frac{M_i C_i}{E_i I_i} \right) \quad (4.3.2)$$

$$\frac{1}{s_{ij}} \cdot \frac{d^2 F_{ij}}{dx^2} = F_{hi} \left(\frac{-1}{E_i A_i} \right) + F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} \right) + F_{jk} \left(\frac{-1}{E_j A_j} \right) - \left(\frac{M_i C_i}{E_i I_i} + \frac{M_j C_j}{E_j I_j} \right) \quad (4.3.3)$$

$$\frac{1}{s_{jk}} \cdot \frac{d^2 F_{jk}}{dx^2} = F_{ij} \left(\frac{-1}{E_j A_j} \right) + F_{jk} \left(\frac{1}{E_j A_j} + \frac{1}{E_k A_k} \right) + F_{kl} \left(\frac{-1}{E_k A_k} \right) - \left(\frac{M_j C_j}{E_j I_j} + \frac{M_k C_k}{E_k I_k} \right) \quad (4.3.4)$$

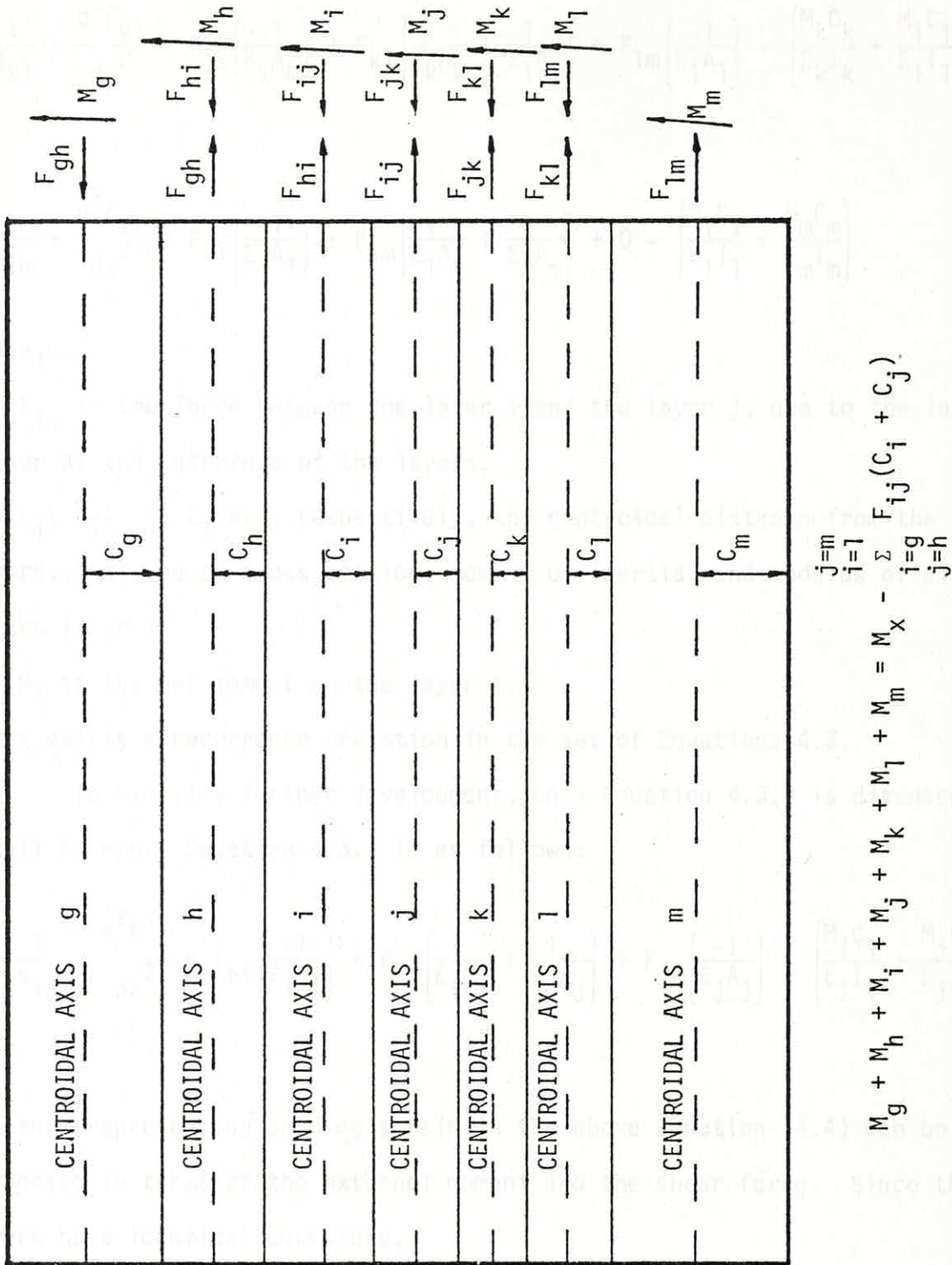


Fig. 4.2. Analysis model for a multilayer system.

$$\frac{1}{s_{k1}} \cdot \frac{d^2 F_{k1}}{dx^2} = F_{jk} \left(\frac{-1}{E_k A_k} \right) + F_{k1} \left(\frac{1}{E_k A_k} + \frac{1}{E_1 A_1} \right) + F_{1m} \left(\frac{-1}{E_1 A_1} \right) - \left(\frac{M_k C_k}{E_k I_k} + \frac{M_1 C_1}{E_1 I_1} \right) \quad (4.3.5)$$

$$\frac{1}{s_{1m}} \cdot \frac{d^2 F_{1m}}{dx^2} = F_{k1} \left(\frac{-1}{E_1 A_1} \right) + F_{1m} \left(\frac{1}{E_1 A_1} + \frac{1}{E_m A_m} \right) + 0 - \left(\frac{M_1 C_1}{E_1 I_1} + \frac{M_m C_m}{E_m I_m} \right) \quad (4.3.6)$$

where,

F_{ij} is the force between the layer i and the layer j , due to the interaction at the interface of the layers.

C_i , A_i , I_i , E_i are, respectively, the centroidal distance from the extreme fibers, the area of cross section, moment of inertia, and modulus of elasticity of the layer i .

M_i is the net moment on the layer i .

There exists a recurrence relation in the set of Equations 4.3.

To simplify further development, only Equation 4.3.3 is discussed in detail herein. Equation 4.3.3 is as follows:

$$\frac{1}{s_{ij}} \cdot \frac{d^2 F_{ij}}{dx^2} = F_{hi} \left(\frac{-1}{E_i A_i} \right) + F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} \right) + F_{jk} \left(\frac{-1}{E_j A_j} \right) - \left(\frac{M_i C_i}{E_i I_i} + \frac{M_j C_j}{E_j I_j} \right) \quad (4.4)$$

The term representing bending strain in the above Equation (4.4) can be expressed in terms of the external moment and the shear force. Since the layers have identical curvature,

$$\frac{M}{EI} = \frac{M_g}{E_g I_g} = \dots = \frac{M_i}{E_i I_i} = \dots = \dots = \frac{M_m}{E_m I_m}$$

From Equation 4.1, the above relationship yields,

$$\frac{M_g}{E_g I_g} = \dots = \dots = \frac{M_i}{E_i I_i} = \dots = \frac{M_m}{E_m I_m} = \frac{M_x - \sum_{i=g, j=h}^{i=1, j=m} F_{ij} (C_i + C_j)}{E_g I_g + E_h I_h + \dots + \dots + E_m I_m}$$

where M_x is the moment due to the external load at any section along the length of the diaphragm.

Using the above relationship, Equation (4.4) can be rewritten as

$$\frac{1}{s_{ij}} \cdot \frac{d^2 F_{ij}}{dx^2} = F_{hi} \left(\frac{-1}{E_i A_i} \right) + F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} \right) + F_{jk} \left(\frac{-1}{E_j A_j} \right) - \frac{M_x (C_i + C_j)}{E_g I_g + E_h I_h + \dots + \dots + E_m I_m} + \frac{(C_i + C_j)}{E_g I_g + E_h I_h + \dots + \dots + E_m I_m} \sum_{i=g, j=h}^{j=m, i=1} F_{ij} (C_i + C_j)$$

Collecting terms and rearranging,

$$\begin{aligned} \frac{1}{s_{ij}} \cdot \frac{d^2 F_{ij}}{dx^2} &= F_{gh} (C_g + C_h) \cdot \frac{(C_i + C_j)}{EI} + F_{hi} \left(\frac{-1}{E_i A_i} + (C_h + C_i) \cdot \frac{(C_i + C_j)}{EI} \right) \\ &+ F_{ij} \left(\frac{1}{E_i A_i} + \frac{1}{E_j A_j} + (C_i + C_j) \cdot \frac{(C_i + C_j)}{EI} \right) \\ &+ F_{jk} \left(\frac{-1}{E_j A_j} + (C_j + C_k) \cdot \frac{(C_i + C_j)}{EI} \right) \\ &+ F_{kl} (C_k + C_l) \cdot \frac{(C_i + C_j)}{EI} + F_{lm} (C_l + C_m) \cdot \frac{(C_i + C_j)}{EI} - \frac{M_x (C_i + C_j)}{EI} \quad (4.5) \end{aligned}$$

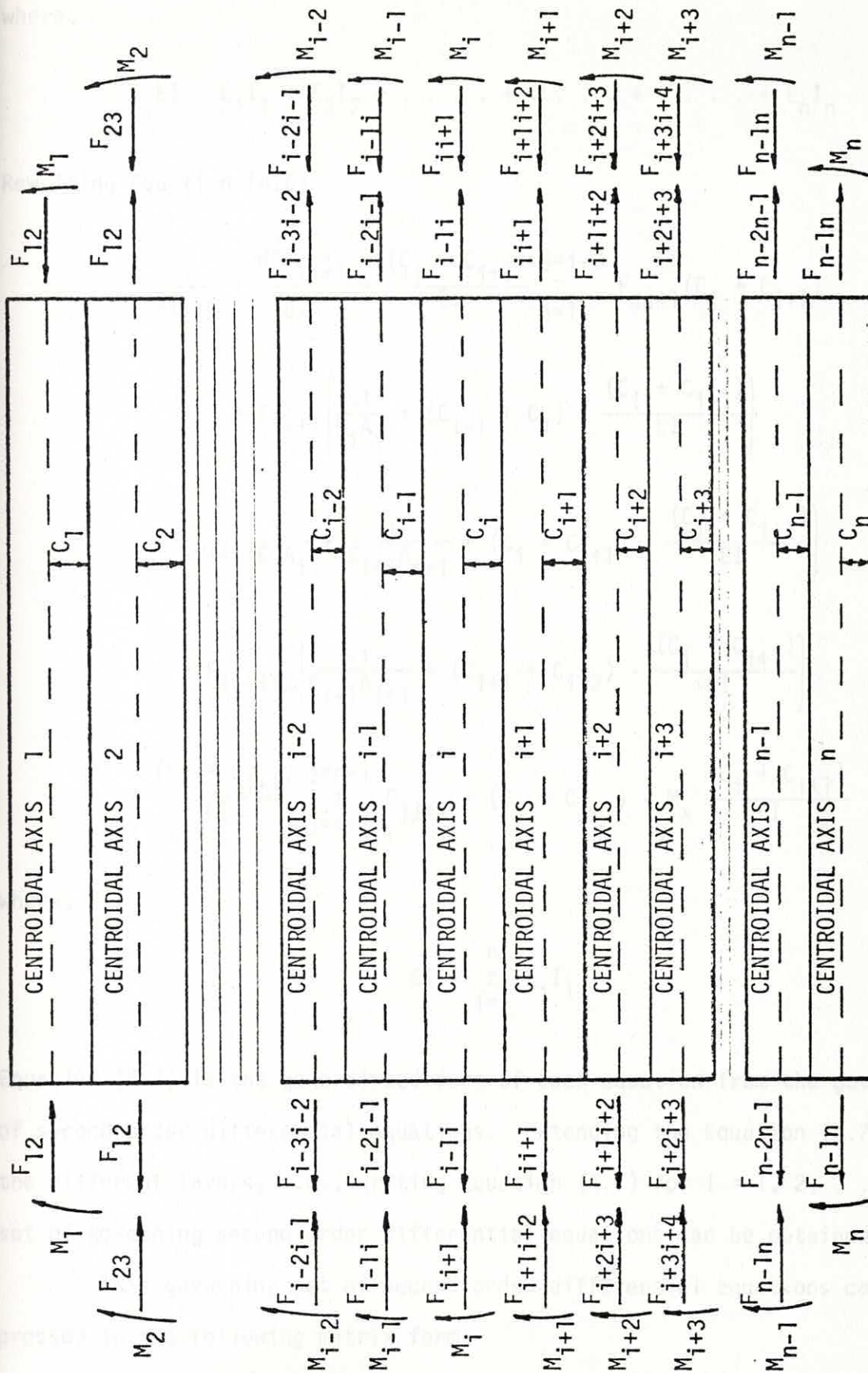
where $EI = E_g I_g + E_h I_h + \dots + \dots + E_m I_m$.

Equation (4.5) is one of the governing equations. To express it in a better form, refer to Figure 4.3. In this case, there are 'n' layers. A governing equation for the interaction between i^{th} layer and $(i+1)^{\text{th}}$ layer can be written from Equation (4.5) by merely substituting the following:

$$g = 1, h = 2, \dots, i = i, j = i+1, \dots, l = n-1, m = n$$

therefore,

$$\begin{aligned} \frac{1}{s_{ii+1}} \frac{d^2 F_{ii+1}}{dx^2} &= F_{12}(C_1 + C_2) \cdot \frac{(C_i + C_{i+1})}{EI} + F_{23}(C_2 + C_3) \cdot \frac{(C_i + C_{i+1})}{EI} \\ &+ \dots + \dots + \dots + F_{i-2i-1} \cdot (C_{i-2} + C_{i-1}) \cdot \frac{(C_i + C_{i+1})}{EI} \\ &+ F_{i-1i} \left(\frac{-1}{E_i A_i} + (C_{i-1} + C_i) \cdot \frac{(C_i + C_{i+1})}{EI} \right) \\ &+ F_{ii+1} \left(\frac{1}{E_i A_i} + \frac{1}{E_{i+1} A_{i+1}} + (C_i + C_{i+1}) \cdot \frac{(C_i + C_{i+1})}{EI} \right) \\ &+ F_{i+1i+2} \left(\frac{-1}{E_{i+1} A_{i+1}} + (C_{i+1} + C_{i+2}) \cdot \frac{(C_i + C_{i+1})}{EI} \right) \\ &+ F_{i+2i+3} (C_{i+2} + C_{i+3}) \cdot \frac{(C_i + C_{i+1})}{EI} + \dots + \dots \\ &+ \dots + F_{n-1n} (C_{n-1} + C_n) \cdot \frac{(C_i + C_{i+1})}{EI} - \frac{M_x}{EI} (C_i + C_{i+1}) \quad (4.6) \end{aligned}$$



$$\sum_{i=1}^{i=n} M_i = M_x - \sum_{i=1}^{i=n-1} F_{i+1} (C_i + C_{i+1})$$

Fig. 4.3.--Analysis model for n layer system.

where,

$$EI = E_1 I_1 + E_2 I_2 + \dots + \dots + \dots + E_n I_n$$

Rewriting Equation (4.6)

$$\begin{aligned} \frac{1}{s_{ii+1}} \cdot \frac{d^2 F_{ii+1}}{dx^2} &= \frac{(C_i + C_{i+1})}{EI} \sum_{j=1}^{j=i-2} F_{jj+1} (C_j + C_{j+1}) \\ &+ F_{i-1i} \left[\frac{-1}{E_i A_i} + (C_{i-1} + C_i) \cdot \frac{(C_i + C_{i+1})}{EI} \right] \\ &+ F_{ii+1} \left[\frac{1}{E_i A_i} + \frac{1}{E_{i+1} A_{i+1}} + (C_i + C_{i+1}) \cdot \frac{(C_i + C_{i+1})}{EI} \right] \\ &+ F_{i+1i+2} \left[\frac{-1}{E_{i+1} A_{i+1}} + (C_{i+1} + C_{i+2}) \cdot \frac{(C_i + C_{i+1})}{EI} \right] \\ &+ \frac{(C_i + C_{i+1})}{EI} \sum_{j=i+2}^{j=n-1} F_{jj+1} \cdot (C_j + C_{j+1}) - M_x \frac{(C_i + C_{i+1})}{EI} \end{aligned} \quad (4.7)$$

where,

$$EI = \sum_{i=1}^n E_i I_i$$

Equation (4.7) is the generalized form of each equation from the governing set of second order differential equations. Extending the Equation (4.7) to all the different layers, i.e., writing Equation (4.7) for $i = 1, 2, \dots, n$, the set of governing second order differential equations can be obtained.

This governing set of second order differential equations can be expressed in the following matrix form:

$$\begin{pmatrix} \frac{1}{s_{12}} \cdot \frac{d^2 F_{12}}{dx^2} \\ \frac{1}{s_{23}} \cdot \frac{d^2 F_{23}}{dx^2} \\ \frac{1}{s_{ii+1}} \cdot \frac{d^2 F_{ii+1}}{dx^2} \\ \frac{1}{s_{n-1n}} \cdot \frac{d^2 F_{n-1n}}{dx^2} \end{pmatrix} = [K]_{n-1, n-1} \begin{pmatrix} F_{12} \\ F_{23} \\ F_{ii+1} \\ F_{n-1n} \end{pmatrix} - \frac{M_x}{EI} \begin{pmatrix} C_1 + C_2 \\ C_2 + C_3 \\ C_i + C_{i+1} \\ C_{n-1} + C_n \end{pmatrix}$$

(4.8)

where,

$$k_{ii} = \frac{1}{E_i A_i} + \frac{1}{E_{i+1} A_{i+1}} + (C_i + C_{i+1}) \cdot \frac{(C_i + C_{i+1})}{EI}$$

$$k_{ii-1} = \frac{-1}{E_i A_i} + (C_{i-1} + C_i) \cdot \frac{(C_i + C_{i+1})}{EI}$$

$$k_{ii+1} = \frac{-1}{E_{i+1} A_{i+1}} + (C_{i+1} + C_{i+2}) \cdot \frac{(C_i + C_{i+1})}{EI}$$

$$k_{ij} = (C_j + C_{j+1}) \cdot \frac{(C_i + C_{i+1})}{EI}$$

if $|i-j| > 1$.

The governing set of equations (Eq. 4.8) is a set of second order linear differential equations.

The coefficient matrix $[K]$ is a symmetric matrix. For a system of n layers, the $[K]$ matrix will be of order $n-1$.

M_x is the external bending moment function. Applying appropriate boundary conditions, the governing set of Equations (4.8) can be solved to find out the shear forces at interfaces of layers in contact.

Thus, a governing set of second order differential equations (Eq. 4.8) to analyze a diaphragm which consists of several glued layers is developed. This does not account for the openings in a diaphragm and also, for the discontinuity in any of the layers.

The mathematical approach and the computer program developed to solve the governing set of second order differential equations (Eq. 4.8) is discussed in the next chapter.

$$\begin{aligned}
 & \frac{1}{E_1 I_1} \frac{d^2 w_1}{dx^2} = \frac{1}{E_2 I_2} \frac{d^2 w_2}{dx^2} = \dots = \frac{1}{E_n I_n} \frac{d^2 w_n}{dx^2} \\
 & \frac{1}{E_1 I_1} \frac{d^2 w_1}{dx^2} = \frac{1}{E_2 I_2} \frac{d^2 w_2}{dx^2} = \dots = \frac{1}{E_n I_n} \frac{d^2 w_n}{dx^2} = \frac{1}{EI} \frac{d^2 w}{dx^2} \\
 & \frac{1}{E_1 I_1} \frac{d^2 w_1}{dx^2} = \frac{1}{E_2 I_2} \frac{d^2 w_2}{dx^2} = \dots = \frac{1}{E_n I_n} \frac{d^2 w_n}{dx^2} = \frac{1}{EI} \frac{d^2 w}{dx^2} \quad (4.8)
 \end{aligned}$$

where

$$\frac{1}{EI} = \frac{1}{E_1 I_1} = \frac{1}{E_2 I_2} = \dots = \frac{1}{E_n I_n}$$

CHAPTER 5

APPLICATION OF THE FINITE DIFFERENCE TECHNIQUE

The set of governing second-order linear differential equations for a multi-layer system was developed in Chapter 4. In order to find the forces between the different layers, these equations are to be solved simultaneously for a known load on the diaphragm and subjected to the known boundary conditions. The mathematical set-up to solve these equations, which is based upon the finite difference method, is discussed in this chapter.

The set of equations for a n -layer system, developed in Chapter 4 is recalled below.

$$\begin{pmatrix} \frac{1}{s_{12}} \cdot \frac{d^2 F_{12}}{dx^2} \\ \frac{1}{s_{ii+1}} \cdot \frac{d^2 F_{ii+1}}{dx^2} \\ \frac{1}{s_{n-1n}} \cdot \frac{d^2 F_{n-1n}}{dx^2} \end{pmatrix}_{n-1,1} = [K]_{n-1,n-1} \begin{pmatrix} F_{12} \\ F_{ii+1} \\ F_{n-1n} \end{pmatrix}_{n-1,1} - \frac{M_x}{EI} \begin{pmatrix} C_1 + C_2 \\ C_i + C_{i+1} \\ C_{n-1} + C_n \end{pmatrix}_{n-1,1} \quad (4.8)$$

where

$$k_{ii} = \frac{1}{E_i A_i} + \frac{1}{E_{i+1} A_{i+1}} + \frac{(C_i + C_{i+1})^2}{EI}$$

$$k_{ii-1} = \frac{-1}{E_i A_i} + \frac{(C_{i-1} + C_i) \cdot (C_i + C_{i+1})}{EI}$$

$$k_{ii+1} = \frac{-1}{E_{i+1}A_{i+1}} + \frac{(C_i + C_{i+1}) \cdot (C_{i+1} + C_{i+2})}{EI}$$

$$k_{ij} = \frac{(C_i + C_{j+1}) \cdot (C_i + C_{i+1})}{EI}$$

if $|i-j| > 1$.

M_x = bending moment due to the load on the diaphragm. Thus, the governing differential equation for the force in i th and $(i+1)$ th layer in an n layer system can be written as

$$\frac{1}{s_{ii+1}} \cdot \frac{d^2 F_{ii+1}}{dx^2} = \sum_{j=1}^{n-1} k_{ij} \cdot F_{jj+1} - \frac{M_x}{EI} \cdot (C_i + C_{i+1}) \quad (5.1)$$

If M_x , the bending moment due to the external load at any section, is along the length of the diaphragm is known, then F_{ii+1} at that section can be calculated.

The matrix $[K]$ and $[s_{ii+1}]_{i=1}^{i=n-1}$, the stiffnesses of the different gluelines, can be computed if the properties of different layers and the glue-lines are known. In order to compute the force F_{ii+1} and the external moment M_x , the boundary conditions must be known. Thus, the problem is to solve the second order differential equation with regards to the boundary conditions.

From the finite difference theory,

$$\left(\frac{d^2 y}{dx^2} \right)_m = \frac{(y)_{m-1} - 2(y)_m + (y)_{m+1}}{h^2} \quad (5.2)$$

where,

m is the point where value of $\left(\frac{d^2 y}{dx^2} \right)$ is to be calculated.

$m-1, m+1$ are the two points located on the boundary of h neighborhood of m (Fig. 5.1).

h is known as finite difference interval.

Now, the coordinate system is oriented such that the left boundary of the diaphragm represents the y -axis and the x -axis lies along the length of the diaphragm (Fig. 5.2). Each glue line and its intersection with the left boundary are treated as the x -axis and its origin, respectively, for the corresponding function of force.

Equation (5.2) can be extended to the L.H.S. of Equation (5.1) as follows:

$$\frac{1}{s_{ii+1}} \cdot \left(\frac{d^2 F_{ii+1}}{dx^2} \right)_m = \frac{1}{s_{ii+1}} \cdot \frac{1}{h^2} [(F_{ii+1})_{m-1} - 2(F_{ii+1})_m + (F_{ii+1})_{m+1}] \quad (5.3)$$

In Equation (5.3), the L.H.S. at a point m along the x -axis is expressed in terms of finite difference. So the R.H.S. of Equation (5.1) must also be evaluated at the same point m .

$$(\text{R.H.S.})_m = \sum_{j=1}^{n-1} k_{ij} (F_{jj+1})_m - \frac{(M_x)_m}{EI} \cdot (C_i + C_{i+1}) \quad (5.4)$$

It should be noted that M_x , the external moment is also a function of x . Thus, from Equation (5.3) and Equation (5.4)

$$\begin{aligned} & \frac{1}{s_{ii+1}} \cdot \frac{1}{h^2} [(F_{ii+1})_{m-1} - 2(F_{ii+1})_m + (F_{ii+1})_{m+1}] \\ & = \sum_{j=1}^{n-1} k_{ij} \cdot (F_{jj+1})_m - \frac{(M_x)_m}{EI} (C_i + C_{i+1}) \end{aligned} \quad (5.5)$$

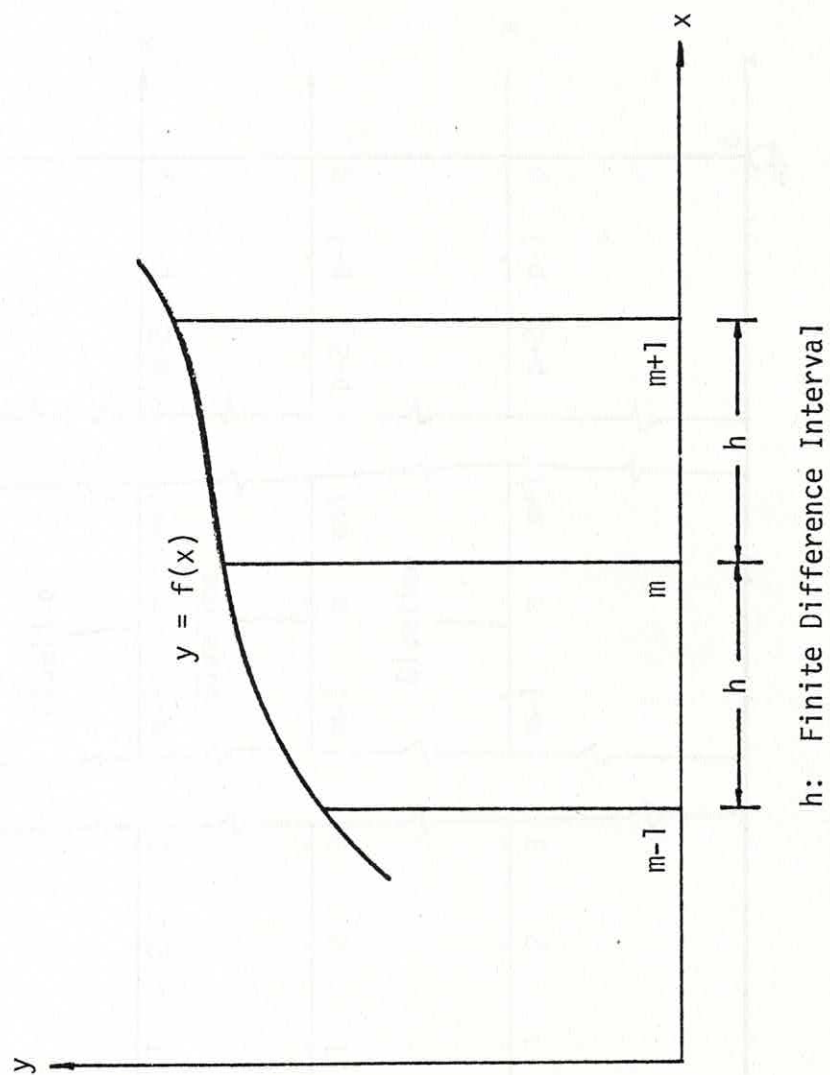


Fig. 5.1.--Finite difference method.

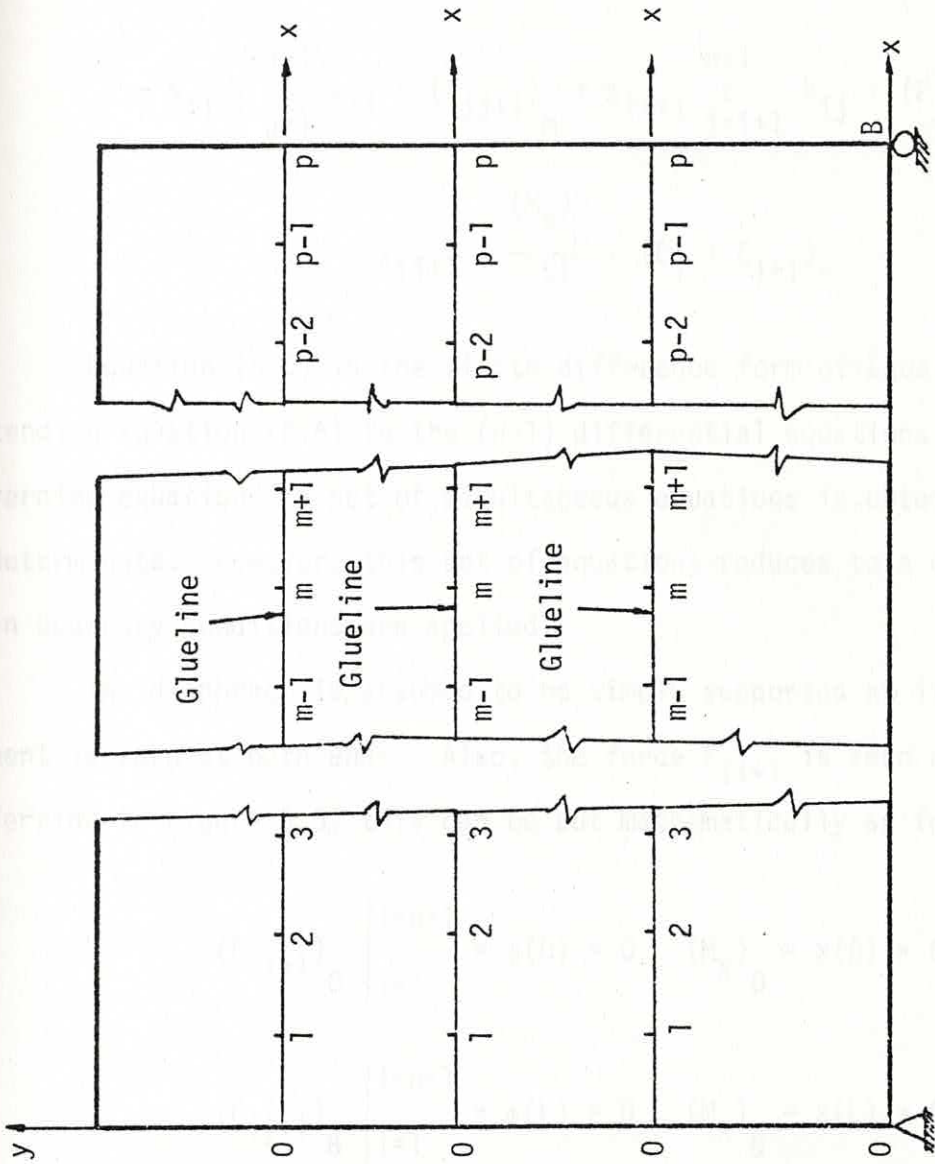


Fig. 5.2.--Four-layer system oriented to apply finite difference method.

Rearranging Equation (5.5)

$$\begin{aligned}
 & \frac{-1}{h^2} (F_{ii+1})_{m-1} + \left(\frac{2}{h^2} + s_{ii+1} \cdot k_{ii} \right) (F_{ii+1})_m - \frac{1}{h^2} (F_{ii+1})_{m+1} \\
 & + s_{ii+1} \sum_{j=1}^{i-1} k_{ij} \cdot (F_{jj+1})_m + s_{ii+1} \sum_{j=i+1}^{n-1} k_{ij} \cdot (F_{jj+1})_m \\
 & = s_{ii+1} \cdot \frac{(M_x)_m}{EI} \cdot (C_i + C_{i+1}) \quad (5.6)
 \end{aligned}$$

Equation (5.6) is the finite difference form of Equation (5.1).

Extending Equation (5.6) to the $(n-1)$ differential equations in the set of governing equations, a set of simultaneous equations is obtained, which is indeterminate. However, this set of equations reduces to a determinate system when boundary conditions are applied.

The diaphragm is assumed to be simply supported at its ends. So, moment is zero at both ends. Also, the force F_{ii+1} is zero at both the ends. Referring to Figure 5.3, this can be put mathematically as follows:

$$(F_{ii+1})_0 \Big|_{i=1}^{i=n-1} = \phi(0) = 0 \quad (M_x)_0 = x(0) = 0$$

$$(F_{ii+1})_B \Big|_{i=1}^{i=n-1} = \phi(L) = 0 \quad (M_x)_B = x(L) = 0$$

Thus, for $m = 1$, Equation (5.1) would reduce as follows:

$$\begin{aligned}
 & \left(\frac{2}{h^2} + s_{ii+1} \cdot k_{ii} \right) (F_{ii+1})_1 - \frac{1}{h^2} (F_{ii+1})_2 + s_{ii+1} \sum_{j=1}^{i-1} k_{ij} \cdot (F_{jj+1})_1 \\
 & + s_{ii+1} \sum_{j=i+1}^{n-1} k_{ij} (F_{jj+1})_m = s_{ii+1} \frac{(M_x)_1}{EI} (C_i + C_{i+1}) \quad (5.7)
 \end{aligned}$$

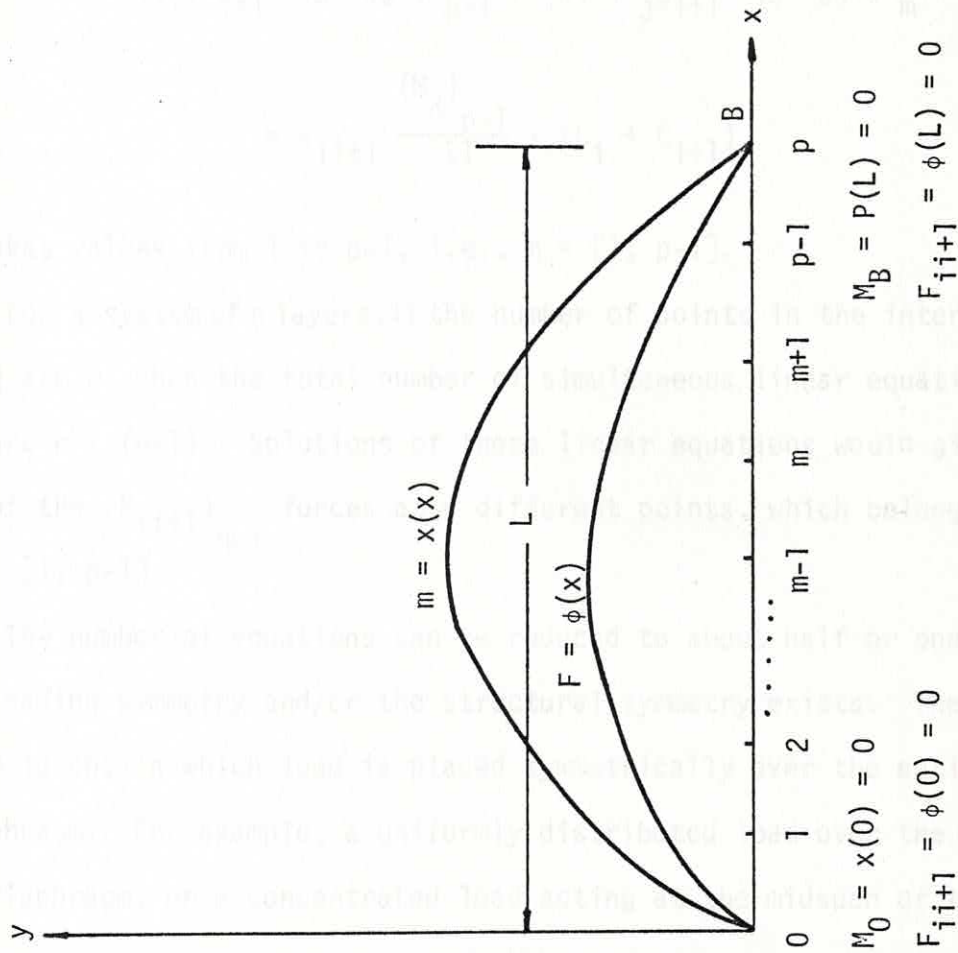


Fig. 5.3.--Boundary conditions for a diaphragm simply supported at both the ends.

For $m = p-1$

$$\begin{aligned}
 & \frac{-1}{h^2} \cdot (F_{ii+1})_{p-2} + \left(\frac{2}{h^2} + s_{ii+1} \cdot k_{ii} \right) (F_{ii+1})_{p-1} \\
 & + s_{ii+1} \sum_{j=1}^{i-1} k_{ij} (F_{jj+1})_{p-1} + s_{ii+1} \sum_{j=i+1}^{n-1} k_{ij} (F_{jj+1})_m \\
 & = s_{ii+1} \frac{(M_x)_{p-1}}{EI} \cdot (C_i + C_{i+1}) \quad (5.8)
 \end{aligned}$$

So, m takes values from 1 to $p-1$, i.e., $m = [1, p-1]$.

For a system of n layers, if the number of points in the interval $[1, p-1]$ are r , then the total number of simultaneous linear equations to be solved are $r \cdot (n-1)$. Solutions of these linear equations would give the values of the $\{F_{ii+1}\}_{n-1}$ forces at r different points, which belong to the interval $[1, p-1]$.

The number of equations can be reduced to about half or one-fourth, if the loading symmetry and/or the structural symmetry exists. The loading symmetry is one in which load is placed symmetrically over the entire span of the diaphragm. For example, a uniformly distributed load over the entire span of the diaphragm, or a concentrated load acting at the midspan of the diaphragm, etc.

The structural symmetry in a diaphragm is with respect to the arrangement of the different courses and the different gluelines. If the portion of a diaphragm below its centroidal axis is a mirror image of the portion of the diaphragm above the centroidal axis in light of the arrangement of layers and the gluelines, then the diaphragm is treated as structurally symmetric. In such cases,

$$|F_{12}| = |F_{n-1n}|$$

$$|F_{23}| = |F_{n-2,n-1}|$$

$$\vdots$$

So, the number of unknowns would reduce to half.

The loading symmetry implies

$$(M_x)_{m-1} = (M_x)_{m+1}$$

So the points in the interval $[1, p-1]$ would reduce to $\frac{r+1}{2}$, since

$$F_{i-1} = F_{i+1}$$

If h is the interval and L is the span, then

$$r = \frac{L}{h} - 1$$

Thus, it is advantageous to consider the structural symmetry and the loading symmetry so that the number of linear equations are reduced.

Thus, the set of governing second order differential equations (Equation 4.8) is reduced to a set of linear simultaneous equations by the finite difference method. Solving these linear equations, the forces can be calculated.

Once the forces are known, the shear flow, shear stress in the glue-lines, etc., the deflection of the diaphragm, the strains and stresses in each layer of a diaphragm can be computed. The relations that would enable the computations are given below.

$$(q_{ii+1})_m = \frac{(F_{ii+1})_{m+1} - (F_{ii+1})_{m-1}}{2h}$$

$$(\tau_{ii+1})_m = \frac{(q_{ii+1})_m}{b_{ii+1}}$$

where, $(q_{ii+1})_m$, $(\tau_{ii+1})_m$ are the shear flow and shear stress in the glue line between the i th layer and the $(i+1)$ th layer, respectively, at the m point along the length of the diaphragm. b_{ii+1} is the width of the glue line.

$$\left(\frac{d^2y}{dx^2}\right)_m = \frac{(M_x)_m - \sum_{j=1}^{n-1} (k_{ij})\{F_{jj+1}\}_m}{EI}$$

where, y is the deflection function.

$$e_{i_t} = \frac{(-F_{ii+1})_m + (F_{i-1i})_m}{E_i A_i} - \frac{\left[(M_x)_m - \sum_{j=1}^{n-1} k_{ij} \cdot \{F_{jj+1}\}_m \right] C_i}{EI}$$

$$e_{i_b} = \frac{(-F_{ii+1})_m + (F_{i-1i})_m}{E_i A_i} + \frac{\left[(M_x)_m - \sum_{j=1}^{n-1} k_{ij} \cdot \{F_{jj+1}\}_m \right] C_i}{EI}$$

$$\sigma_{i_t} = \epsilon_{i_t} \cdot E_i$$

$$\sigma_{i_b} = \epsilon_{i_b} \cdot E_i$$

where, ϵ_{i_t} , σ_{i_t} , ϵ_{i_b} , σ_{i_b} are the maximum strains and the maximum stresses in the top and bottom fibers of the i th layer, respectively.

A computer program DAD--Difference Analysis of Diaphragm--is developed to compute the above (Appendix A and Appendix B).

CHAPTER 6

COMPARISON OF THE THEORETICAL ANALYSIS OF
DIAPHRAGMS WITH EXPERIMENTAL RESULTS

To obtain some insight to the usefulness of the mathematical analysis, the diaphragm 60 ft x 20 ft analyzed experimentally by Johnson [14] was studied using the DAD program. The results of this verification are presented herein.

Johnson's test diaphragm was well detailed in most respects, but the documentation of the connections between the sheathing planks and the boundary members and joists was absent. It was not possible to learn the exact nature of these connections, except that they were nailed, without any adhesive bond. Nail size and spacing could not be determined. The adhesive used to make the bonds between the courses of sheathing plank was a well known brand for which we have information such as shear modulus and shear strength (Scotch-Grip Wood Adhesive No. 5230).

The study was compared with a 100% glued diaphragm. Calculations were based on sixteen courses of 1.5 inch x 15 inch with sheathing plank having an average elastic modulus of 1,200,000 psi (Commercial grade decking panels of Lodgepole pine). The glue line was assumed to be continuous for the full diaphragm length of 60 ft, having two probable values of adhesive shear modulus 75 psi and 100 psi. The large value corresponds to that obtained by Hoyle and Hsu [9, 11] in two different studies. The deflection was computed for uniformly distributed loads along the 60 ft length, corresponding to an end shear of 600 lb/lineal ft. The thickness and width of the glue line is not mentioned

clearly, but by the inquiry thickness was found to be about 3/16 inch, while the width was about 1/2 inch to 3/4 inch. The glue line thickness was thought to be large so that the glue would overflow. As a result, glue line thickness of 1/16 inch to 3/32 inch, which is normally used in practice, was considered. This, in effect, would change the stiffness of the glue line which is defined as:

$$s = \frac{G \cdot b}{t}$$

The measured deflection was 1.1 inch according to Johnson's [14] report.

The experimental diaphragm received added support from the boundary members and the nail couples acting between sheathing and joints. The exact amount of support offered by these parts of the structure is not known, but would not be very large. Also, the experimental diaphragm contained unconnected end joints at intervals in each course, which would reduce the stiffness slightly, but probably not significantly for this type of system. It is, therefore, concluded that the computed result shows good agreement with the available test results. It will be useful to conduct a well instrumented experiment to verify the calculated stresses and deformations, using a diaphragm constructed specifically for this purpose, but that is beyond the scope of this study.

TABLE 6.1.--Values of maximum deflection of the diaphragm for different properties of gluelines.

G	b	t	s	Deflection
psi	inch	inch	psi	inch
75	0.5	0.09375	400.00	1.553
75	0.75	0.09375	600.00	1.09
100	0.75	0.09375	800.00	0.875
75	0.75	0.1875	300.00	1.977
100	0.75	0.1875	400.00	1.553
100	0.75	0.09375	800.00	0.875
100	0.5	0.0625	800.00	0.875
75	0.5	0.0625	600.00	1.09
75	0.75	0.0625	900.00	0.7744

G = shear modulus of the glue (psi).

b = width of the glueline (inch).

t = thickness of the glueline (inch).

s = stiffness of the glueline (psi).

CHAPTER 7

PRESENTATION OF RESULTS, DISCUSSION AND CONCLUSIONS

An elastomeric adhesive bonded diaphragm consists of parallel courses which have about the same properties. It is important to take into consideration problems associated with shrinkage due to seasoning of these courses in place. In his experimental study of a 60 ft x 20 ft diaphragm using 15-inch wide, 1.5-inch thick decking, Johnson [14] observed the failure of the glue-line due to shrinkage. Good commercial decking would be seasoned to 15% maximum moisture content prior to installation. Seasoning in place, after diaphragm construction, would bring the moisture content of the decking down to about 6%. Shrinkage of the decking would set the glue-line in tension. The elastomeric adhesives are capable of only 100% deformation. Thus, though the curing of the glue-line is not affected by moisture content, the glue-line may fail because of too much shrinkage.

Too much shrinkage would demand a thicker glue-line that could withstand the tension deformation perpendicular to its own plane. The thicker the glue-line, the lower its stiffness. As a result, the rigidity of the bond between the parallel courses would decrease. Also, the economic aspect would challenge the utility of the elastomeric adhesive in diaphragm construction.

The solution to the above problem lies in overcoming the shrinkage effect. This can be achieved by using seasoned courses or using smaller courses.

A study of the possible answer to the problems, stated above, and associated areas is described in the following pages.

7.1. Factors Affecting the Behavior of an Elastomeric Adhesive Diaphragm

The behavior of an elastomeric diaphragm is studied in light of the maximum deflection and the maximum strains in the diaphragm. The important factors affecting the behavior of the diaphragm are as follows:

1. The total number of decking courses.

If the total number of decking courses is doubled without changing the size of the diaphragm, each layer is divided into two layers of half the depth. As a result, a rigid bond at the half depth of each layer is replaced by a nonrigid or imperfect bond. This will decrease overall rigidity of the diaphragm.

2. The stiffness of a glueline.

The stiffness of a glueline between various layers affects the overall rigidity of a diaphragm significantly.

3. The properties of the different layers.

It must be emphasized that not only the properties of different layers but the arrangement of the layers of different properties is important and may be worthy of consideration.

7.2. Study of the Factors Affecting the Behavior of a Diaphragm

The computer program DAD--Difference Analysis of Diaphragms--is used to compute the deflection and the strains in an elastomeric adhesive bonded

* Generally, in a diaphragm, properties of different layers are the same.

diaphragm. A diaphragm 60 ft x 20 ft subjected to an end shear of 600 plf is analyzed for the following:

1. Varying stiffness of the gluelines, but the total number of courses remains the same. Total number of courses is selected to be 16.
2. Varying the total number of courses, but the stiffness of the gluelines remains the same.
3. The stiffness of all the gluelines is the same.

The courses are selected to be of lodgepole pine decking with modulus of elasticity of 1,200,000 psi. The thickness of the courses is selected to be 1.5 inch. All the courses as well as all the gluelines have identical respective properties. The end shear of 600 plf is equivalent to a uniformly distributed load of 24,000 pounds.

Figure 7.1 shows the variation of maximum deflection of the diaphragm against the stiffness of the gluelines. s -- the stiffness of the gluelines is given by, $s = \frac{Gb}{t}$, where G is the shear modulus of the glue and b , t are the width and the thickness of the glueline, respectively. When $s = 0$, maximum deflection = 14.4 inches.

In this case, each layer behaves as an individual. Therefore,

$$\text{Load Resisted by Each Layer} = \frac{24,000}{16} = 1,500 \text{ Pounds}$$

$$\Delta_{\text{MAX}} = \frac{5}{384} \frac{(wl)_i l^3}{E_i I_i}$$

where,

Δ_{MAX} is the maximum deflection of the diaphragm

$l = 720$ inch

$E_i = 1,200,000$ psi

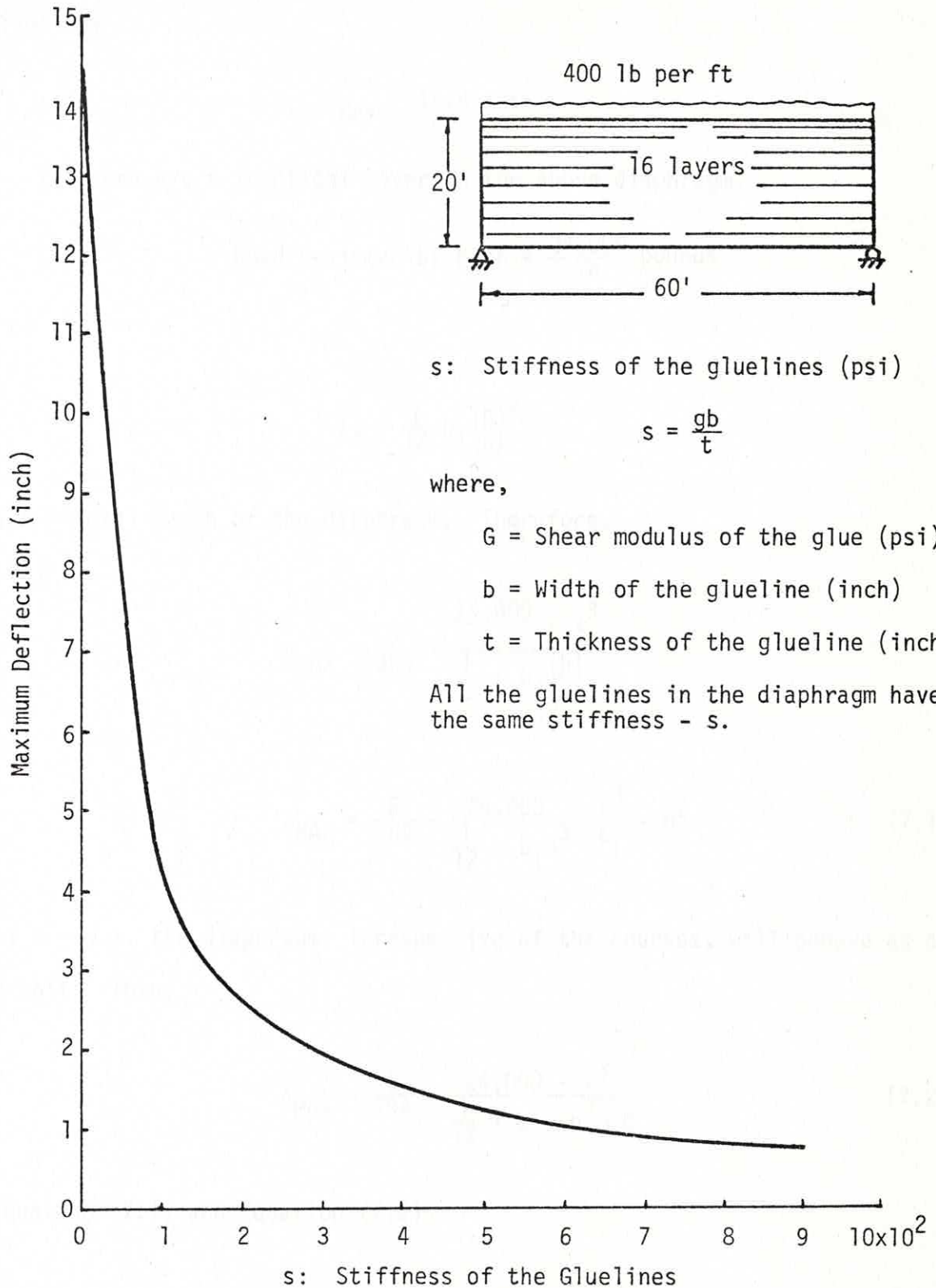


Fig. 7.1.--Maximum deflection versus stiffness of the gluelines.

$$I_i = \frac{1}{12} \cdot b_i h_i^3 = \frac{1}{12} \cdot (1.5)(15)^3 = 421.875 \text{ in}^4$$

substituting,

$$\Delta_{\text{MAX}} = 14.4 \text{ inch}$$

If there are n identical layer in the above diaphragm,

$$\text{Load Resisted by Each} = \frac{24,000}{n} \text{ pounds}$$

Then,

$$I_i = \frac{1}{12} b_i \left(\frac{h}{n}\right)^3$$

where, h = total depth of the diaphragm. Therefore,

$$\Delta_{\text{MAX}} = \frac{5}{384} \cdot \frac{\frac{24,000}{n} \cdot \ell^3}{\frac{1}{12} \cdot b_i \left(\frac{h}{n}\right)^3}$$

$$\Delta_{\text{MAX}} = \frac{5}{384} \cdot \frac{24,000}{\frac{1}{12} \cdot b_i h^3} \frac{\ell^3}{E_i} \cdot n^2 \quad (7.1)$$

but, as $s \rightarrow \infty$, the diaphragm, irrespective of the courses, will behave as a single unit. Then,

$$\Delta_{\text{MAX}} = \frac{5}{384} \frac{24,000 \cdot \ell^3}{\frac{1}{12} \cdot b \cdot h^3 \cdot E} \quad (7.2)$$

from Equation (7.1) and Equation (7.2)

$$(\Delta_{\text{MAX}})_{s=0} = (\Delta_{\text{MAX}})_{s=\infty} n^2 \quad (7.3)$$

This is a relationship which will be discussed in detail later.

Figure 7.1 shows that

1. Even a stiffness of gluelines as low as 100 psi effectively reduces the maximum deflection to 4.56 inches.
2. A stiffness of gluelines of 900 psi, which is practical using available elastomeric wood adhesives, results in the maximum deflection of 0.77 inch. Study by Hoyle and Hsu [9] reveals that the repeated exposure to service loads may decrease the shear modulus of an elastomeric adhesive by 10 to 15 percent. In such a case, from Figure 7.1, the deflection would increase to about 0.9 inch since the stiffness of the gluelines would be about 765 psi.
3. The slope of the curve sharply drops down from $s = 0$ to $s = 100$ psi. From $s = 500$ psi onwards the slope of the curve steadily decreases until the curve becomes asymptotic to the s -axis when maximum deflection is equal to that given by Equation (7.2). In this case, Equation (7.2) yields maximum deflection of 0.05625 inch.

From Equation (7.2) and Equation (7.3), the following relationship can be written, when $s = 0$

$$(\Delta_{MAX})_{s=0} = (\Delta_{MAX})_{s \rightarrow \infty} n^2$$

As $s \rightarrow \infty$

$$(\Delta_{MAX})_{s=0} = (\Delta_{MAX})_{s \rightarrow \infty} n^0$$

So, it can be concluded that as 's' increases from zero to a higher value

$$\Delta_{MAX} = \alpha \cdot n^\beta \quad (7.4)$$

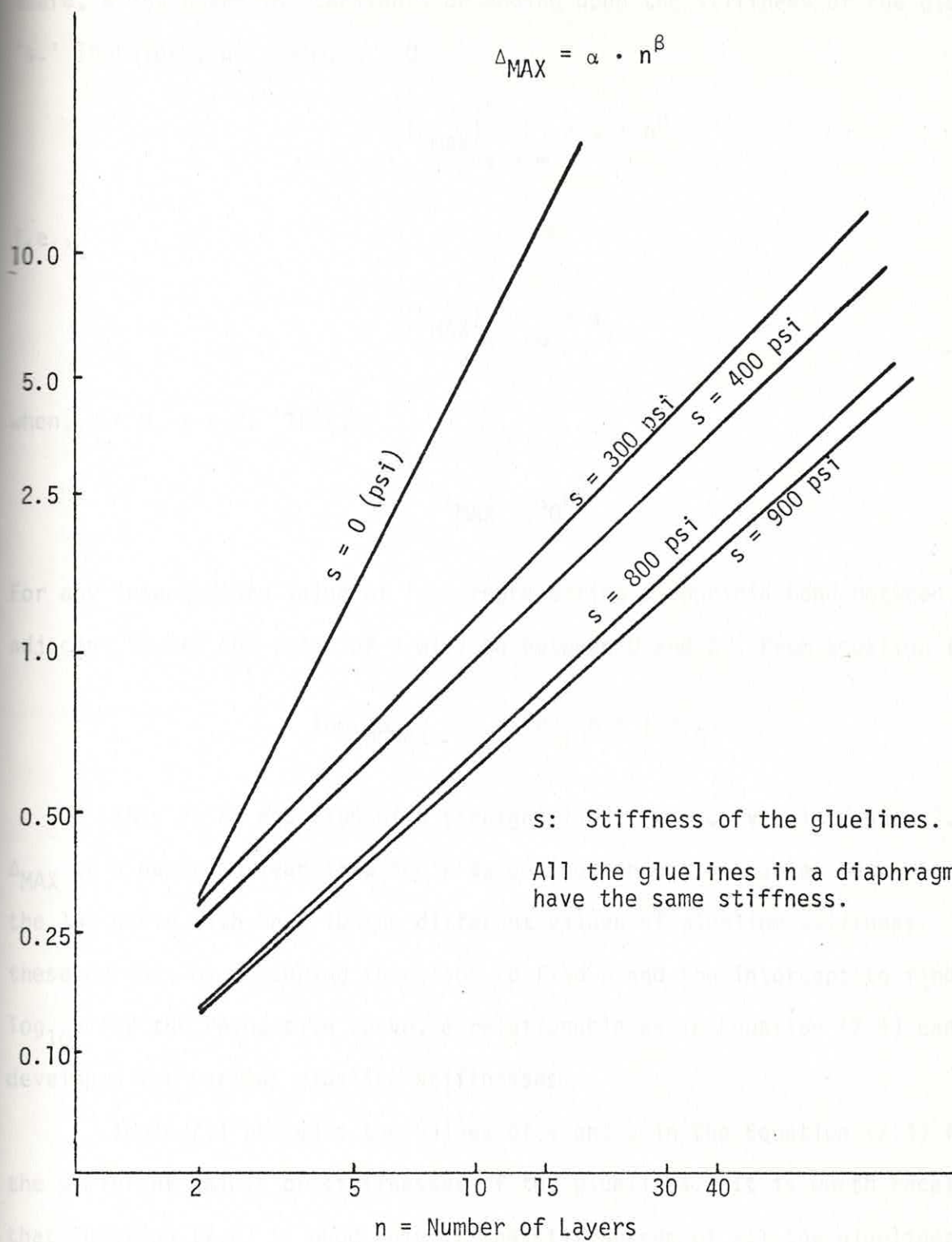


Fig. 7.2.--Maximum deflection versus number of layers in a diaphragm.

where, α and β are the constants depending upon the stiffness of the glueline 's.' Therefore, as $s \rightarrow \infty$, $\beta = 0$

$$(\Delta_{MAX})_{s \rightarrow \infty} = \alpha \cdot n^0$$

i.e.,

$$(\Delta_{MAX})_{s \rightarrow \infty} = \alpha_0$$

when, $s = 0$, $\beta = 2$. Then,

$$\Delta_{MAX} = \alpha_0 n^2 \quad (7.5)$$

For any intermediate value of 's,' representing a nonrigid bond between the adjacent layers the value of β will be between 0 and 2. From Equation (7.4)

$$\log_{10} \Delta_{MAX} = \beta \cdot \log_{10} n + \log_{10} \alpha \quad (7.6)$$

This is an equation of a straight line. The curves in Figure 7.2 show Δ_{MAX} as a dependent variable and n as an independent variable, both plotted on the log scale with base 10 for different values of glueline stiffness. From these curves, by measuring the slope to find β and the intercept to find $\log_{10} \alpha$ for the respective curve, a relationship as in Equation (7.4) can be developed for various glueline stiffnesses.

Table 7.1 presents the values of α and β in the Equation (7.4) for the different values or stiffnesses of the gluelines. It is worth recalling that Equation (7.4) is good only if the stiffnesses of all the gluelines in a diaphragm are equal, at the same time all the layers have the same material and geometric properties. Results for the verification of Equation (7.4) could be observed in Table 7.2.

TABLE 7.1.--Values of α and β for different stiffnesses of the gluelines.

s psi	α	β
0	0.5625	2
300	0.10096	1.07285
400	0.10500	0.97160
800	0.06268	0.94326
900	0.05705	0.94069
∞	0.05625	0

TABLE 7.2.--Verification of the equation, $\Delta_{MAX} = \alpha \cdot n^{\beta}$.

s psi	n	Δ_{MAX} inch		Absolute Relative Error %*
		From Computer Analysis	From Eq. 7.6	
300	24	3.022	3.0543	1.0 %
400	24	2.344	2.3025	1.77 %
800	24	1.260	1.2561	0.3 %
900	20	0.9549	0.9553	0.04 %
900	32	1.4930	1.4864	0.40 %

NOTE: Absolute Relative Error = $\frac{|\text{Error}|}{\Delta_{MAX} \text{ inch from Computer Analysis}}$

The relationship for a glueline stiffness of 900 psi is checked with one of the test results from Johnson [13]

Analytical Value of $\Delta_{MAX} = 1.0$ inch

Experimental Value = 0.76 inch

The difference between the deflections is explained as the influence of diaphragm boundary members and beams which provide a structural effect not accounted for by this layered system analysis. These components increase the stiffness of the system as might be expected and their effect could be introduced.

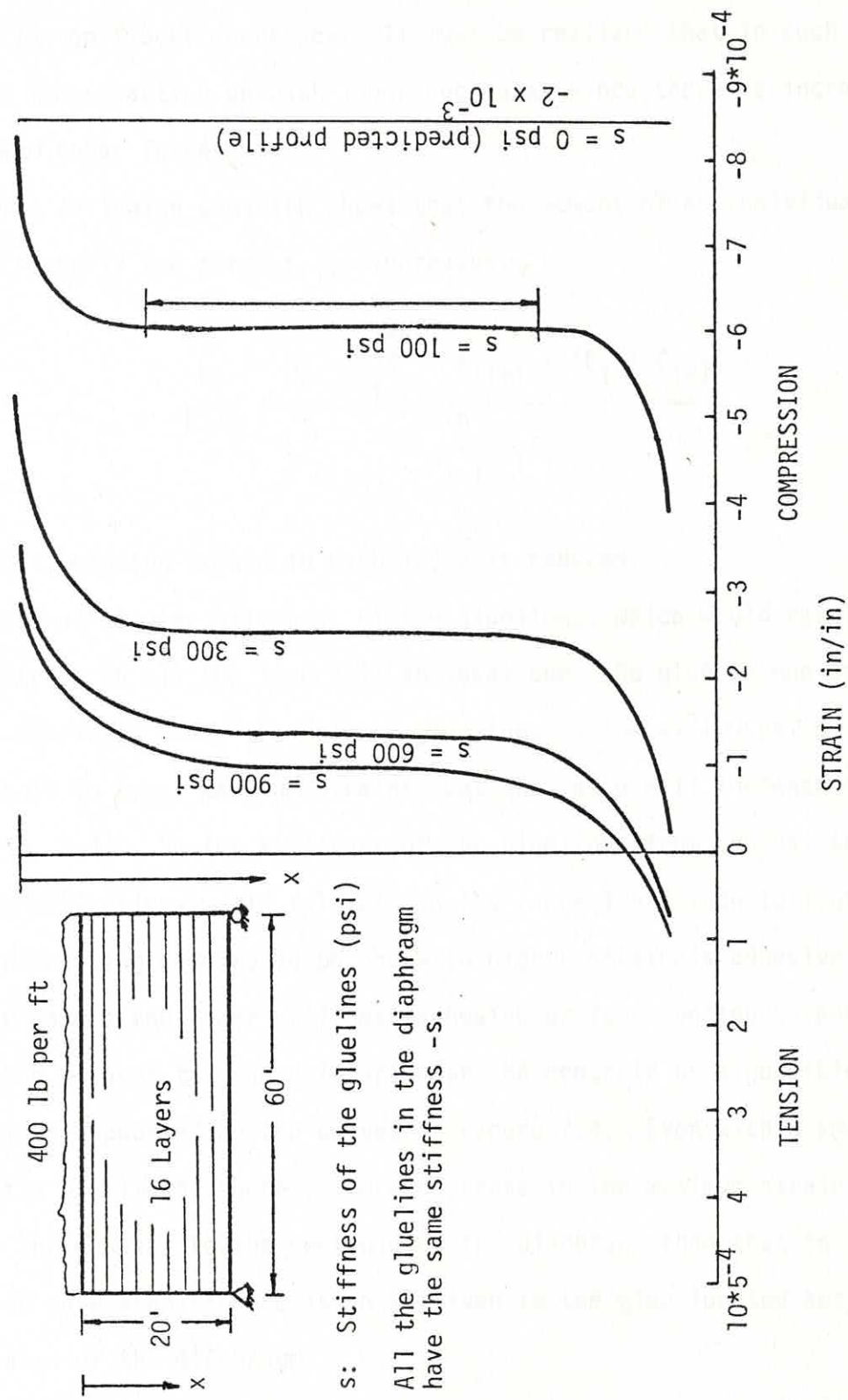
From the above relationships (7.4), it is possible to select the best combination of stiffness of gluelines and number of layers in a diaphragm for an allowable value of maximum deflection.

7.3. Effect of Glueline Stiffness on Strain in Each Layer

The strain variation pertaining to the stiffness of the gluelines is a complex problem.

The curves in Figure 7.3 show the maximum strain in the top fiber of each layer for various stiffnesses of the gluelines. s represents stiffness of a glueline. When $s = 0$, i.e., each layer acts as an individual, maximum strain in each layer would be the same, as shown by the straight-line. When s is 100 psi, the maximum strain in the top fibers* becomes constant over a certain depth, after initial variation. This 'constant strain' zone does not

*The variation in the maximum bottom fiber strain in each layer will be an inverted mirror image of Figure 7.3.



s: Stiffness of the glue-lines (psi)
 All the glue-lines in the diaphragm have the same stiffness -s.

Fig. 7.3.--Profile of maximum strain in the top fibers of each layer in the diaphragm for different glue-line stiffnesses.

appear as stiffness of the gluelines goes on increasing. As the stiffness of the gluelines increases, the axial strain increases (Fig. 3.7), and the compression in top fibers decreases. It must be realized that in such cases the resultant moment acting on each layer decreases since there is increase in the magnitude of shear forces.

The following equation shows that the moment of an individual layer would decrease if the force F_{ii+1} increases.

$$\frac{M_i}{E_i I_i} = \frac{M_x - \sum_{i=1}^{i=n-1} F_{ii+1} \cdot (C_i + C_{i+1})}{\sum_{i=1}^n E_i I_i}$$

As a result, bending strain in each layer is reduced.

The particular stiffness of the gluelines, which would result in equal maximum strain in all the layers is an ideal one. No glue or one causing a value of stiffness $s = \frac{G \cdot b}{t}$ - of the gluelines as low as 100 psi or even less, would result in equal maximum strains, but this also will increase the deflections (Fig. 7.1). So for stiffness of the gluelines from 300 psi to 600 psi, resulting deflections would fall within the range 1.977 inch to 1.098 inch. But an interesting case would be one with higher stiffness adhesive towards the outer layers and lower stiffness adhesive or less continuous adhesive or no adhesive towards the inner layers near the centroid as a possible solution. This fact is supported by the curves in Figure 7.4. Even with a small stiffness of the gluelines, there is more decrease in the maximum strain in layer 8, which is the nearest to the centroid of the diaphragm than that in the topmost layer. So more significance is to be given to the glue located away from the neutral axis of the diaphragm.

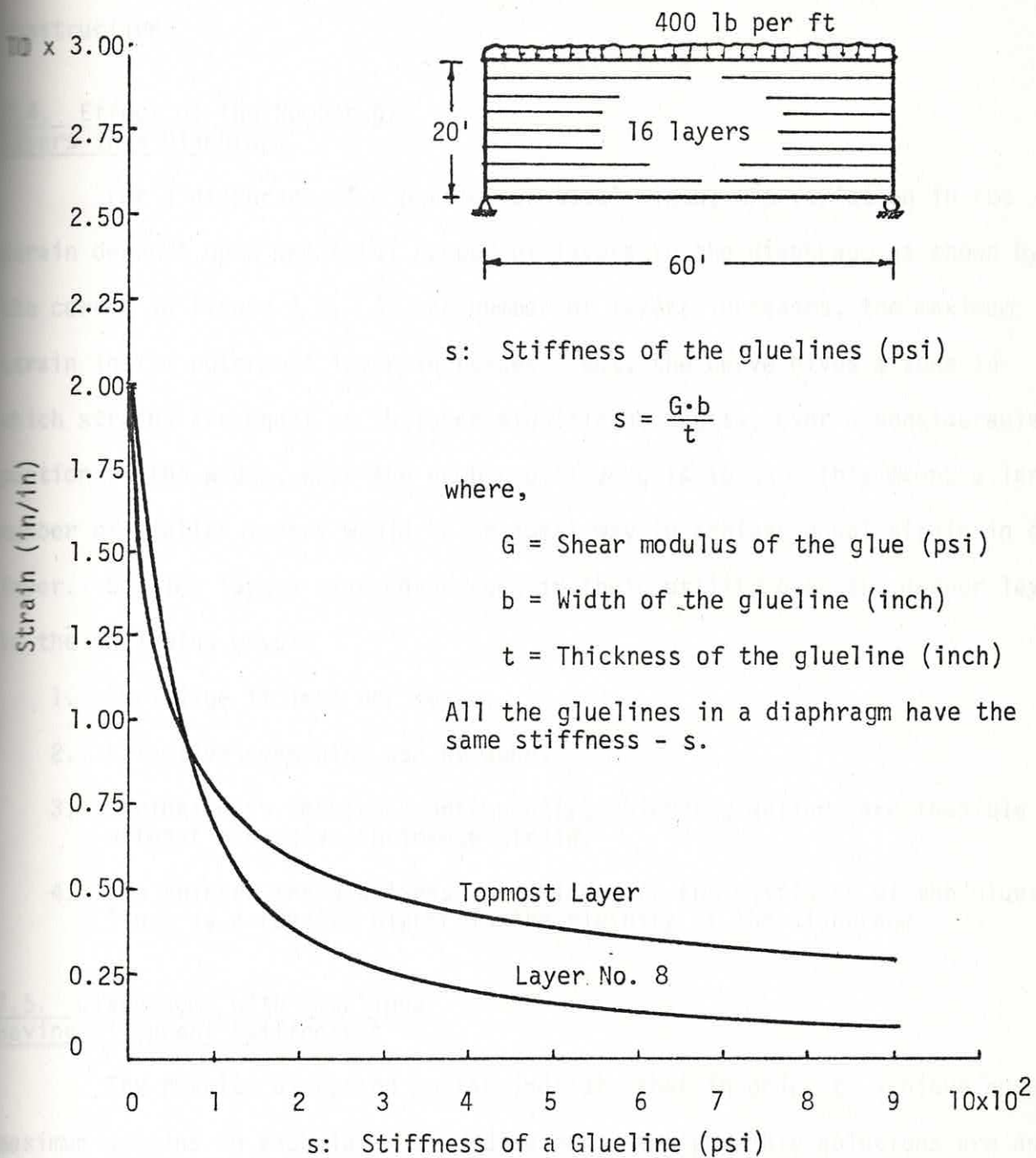


Fig. 7.4.--The maximum strain in the top fibers of the topmost layer and layer No. 8. The stiffness of the gluelines is s.

The constant strain zone when stiffness of glueline is 100 psi may be looked upon as a zone of no glue. This could lead to economies in diaphragm construction.

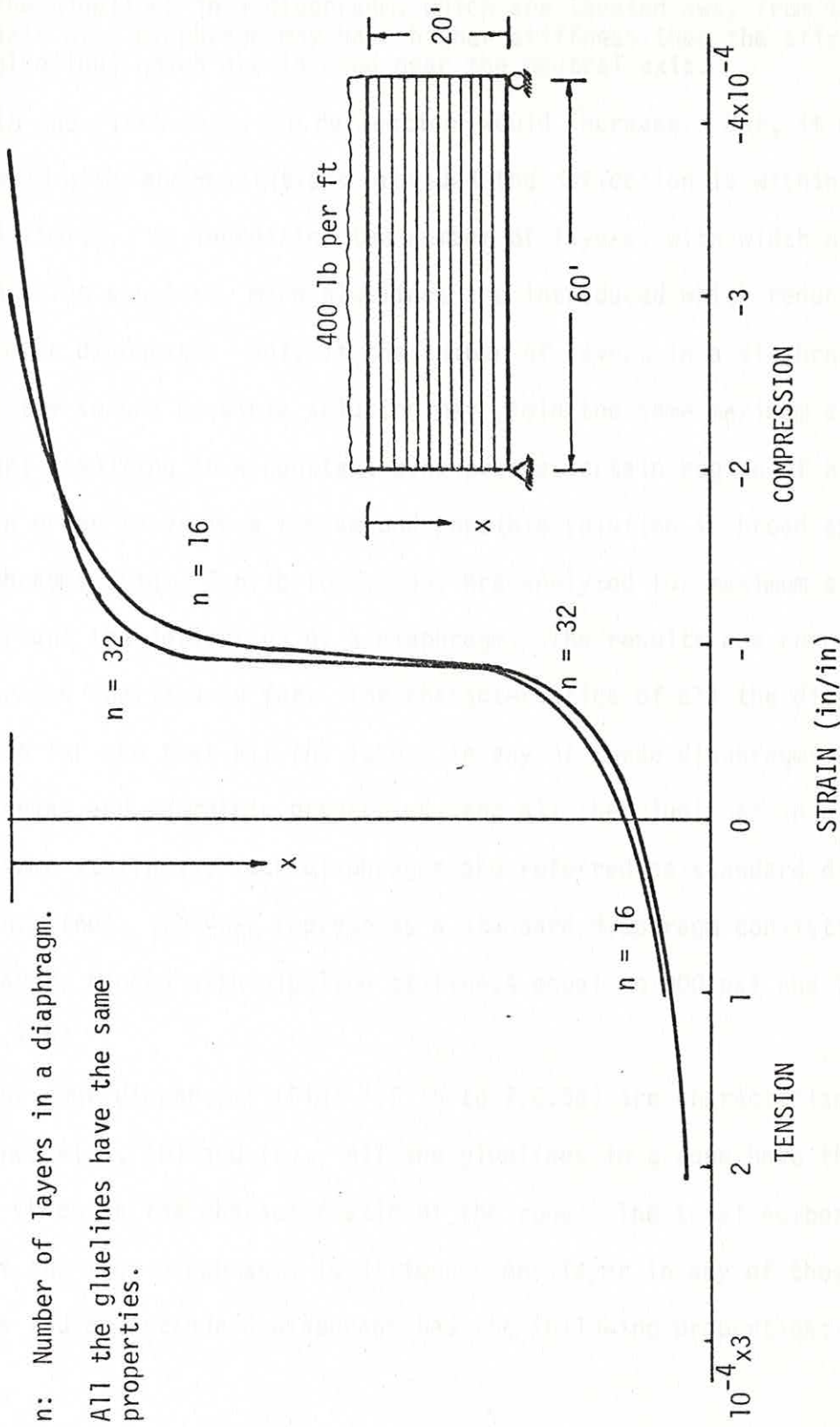
7.4. Effect of the Number of Layers in a Diaphragm

For a diaphragm of a particular total width, the variation in the strain depends upon the total number of layers in the diaphragm as shown by the curves in Figure 7.5. As the number of layers increases, the maximum strain in the outermost layer increases. But, the curve gives a zone in which strains are equal up to three significant digits, over a considerable portion of the width, when the number of layers is large. This means a large number of smaller layers would be an ideal way to achieve equal strain in each layer. Smaller layers have advantages in their utility over the deeper layers in the following ways:

1. Shrinkage is less per layer.
2. Effective seasoning can be done.
3. Shrinkage is reduced, consequently, thinner gluelines are feasible without excessive shrinkage strain.
4. The thinner the gluelines, the higher is the stiffness of the glue-line; as a result, higher is the rigidity of the diaphragm.

7.5. Diaphragms with Gluelines Having Different Stiffnesses

The results discussed so far indicate that in order to achieve equal maximum strains in each layer of a diaphragm, the possible solutions are as follows:



n: Number of layers in a diaphragm.

All the gluelines have the same properties.

Fig. 7.5.--Profile of maximum strain in the top fibers of each layer for different number of layers in a diaphragm.

1. The diaphragm may consist of a large number of smaller layers.
2. The gluelines in a diaphragm, which are located away from the neutral axis of a diaphragm may have higher stiffness than the stiffness of gluelines which are located near the neutral axis.

In the first case, the deflection would increase. But, it may be possible to employ the above solutions provided the deflection is within the allowable limits. In increasing the number of layers, with width of the diaphragm being constant, more gluelines are introduced which reduce the rigidity of a diaphragm. But, if the number of layers in a diaphragm remains the same, the second possible solution may yield the same maximum strain in each layer; resulting in a constant zone over a certain region of a diaphragm.

In order to explore the second possible solution in broad aspect, four diaphragms (Figs. 7.6.1b to 7.6.3a) are analyzed for maximum strains in each layer and the deflection of a diaphragm. The results are compared with the diaphragms analyzed so far. The characteristics of all the diaphragms analyzed so far are that all the layers in any of these diaphragms have identical material and geometric properties, and all the gluelines in a diaphragm have the same stiffness; such diaphragms are referred as standard diaphragms henceforth. Thus, 16^{Std}_{900} represents a standard diaphragm consisting of sixteen layers bonded with glueline stiffness equal to 900 psi and likewise (Fig. 7.6.1.a).

The four diaphragms (Fig. 7.6.1b to 7.6.3a) are characterized by different zones--(a), (b) and (c). All the gluelines in a zone have the same stiffness which is the characteristic of the zone. The total number of layers in each of the four diaphragms is sixteen. Any layer in any of these four diaphragms and any standard diaphragm has the following properties:

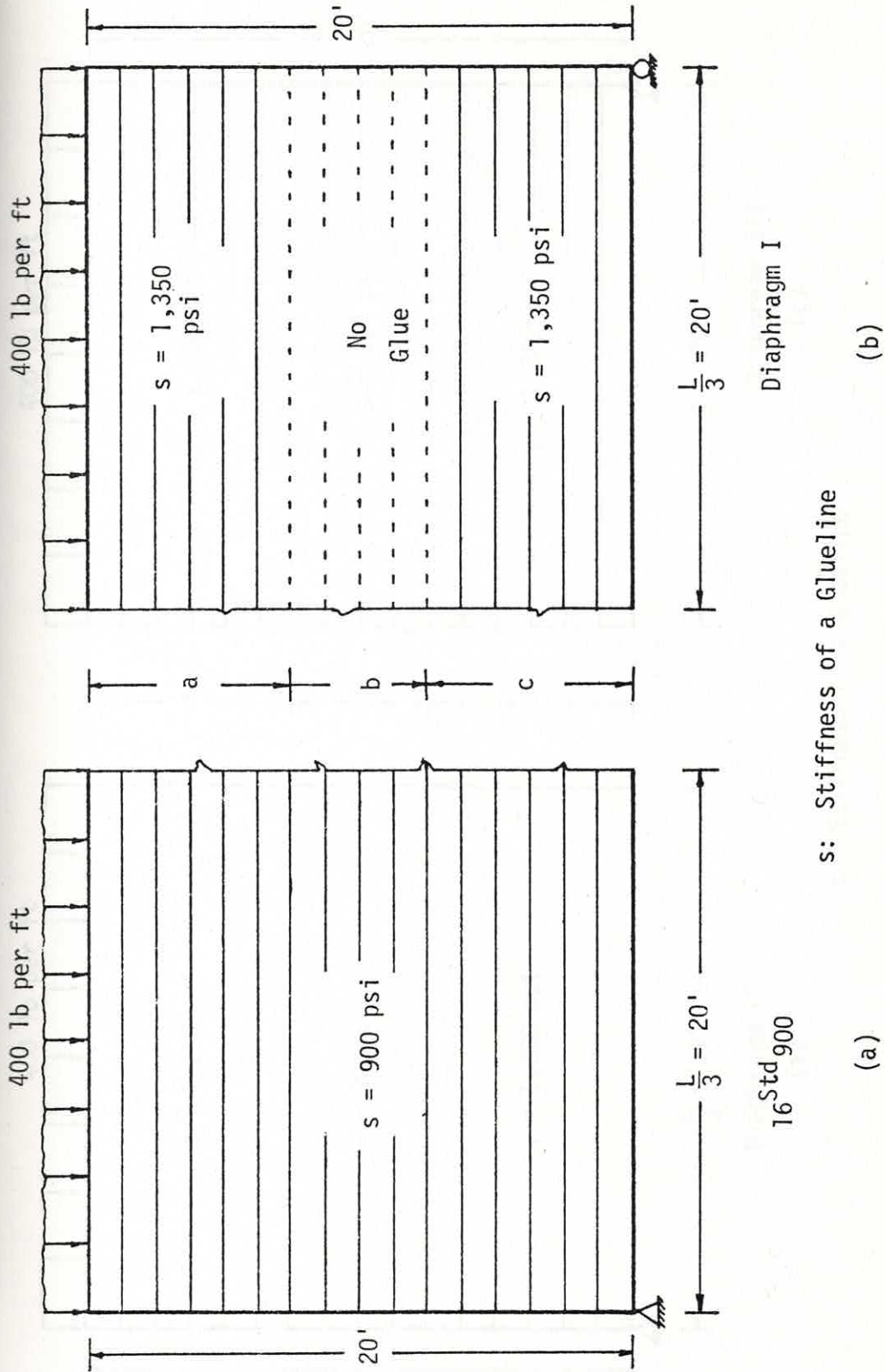


Fig. 7.6.1.--16Std 900 and Diaphragm I with different stiffnesses of the glue lines.

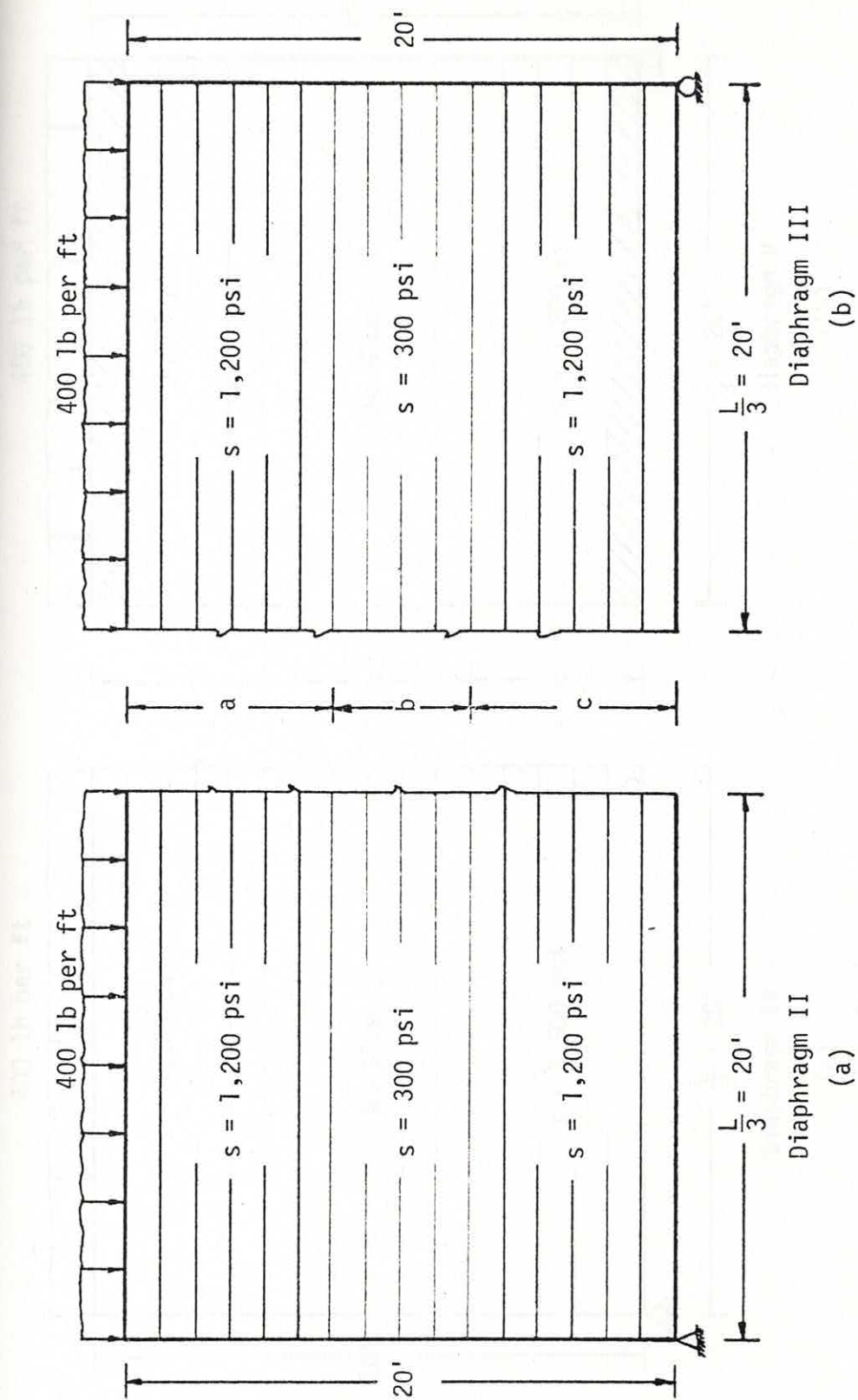


Fig. 7.6.2.--Diaphragm II and III with different stiffnesses of the glueline-- s .

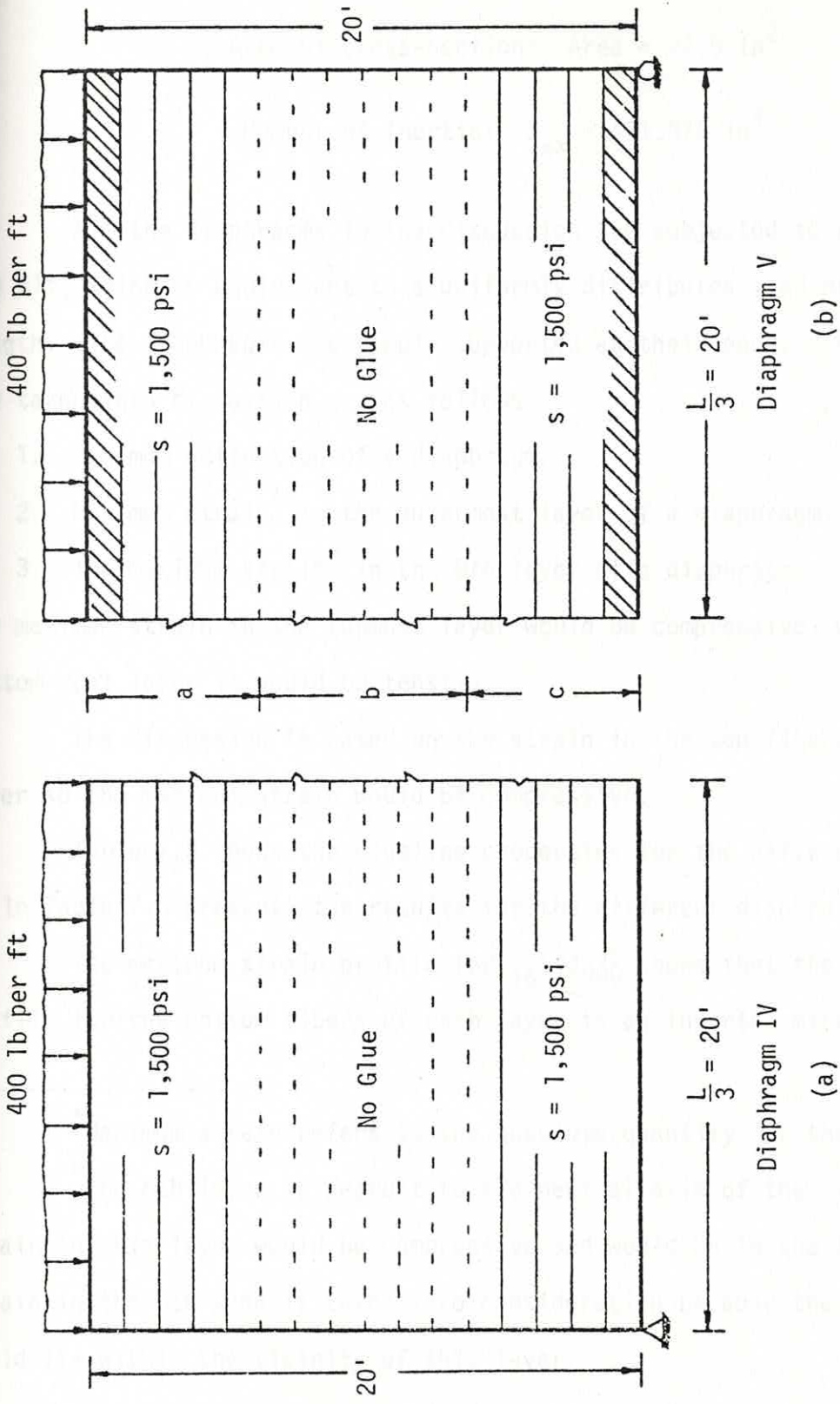


Fig. 7.6.3.--Diaphragm IV and V with different stiffnesses of the glue--s.

Modulus of Elasticity: $E = 1.2 * 10^6$ psi

Area of Cross-Section: $Area = 22.5$ in²

Moment of Inertia: $I_{xx} = 421.875$ in⁴

All the diaphragms in the discussion are subjected to an end shear of 600 plf, which is equivalent to a uniformly distributed load of 400 lb per ft length. The diaphragms are simply supported at their ends. The results which are taken into discussion are as follows:

1. Maximum deflection of a diaphragm.
2. Maximum strain* in the outermost layer of a diaphragm.
3. The maximum strain* in the 8th layer of a diaphragm.

The maximum strain in the topmost layer would be compressive, while in the bottom-most layer it would be tensile.

The discussion is based on the strain in the top fibers of the topmost layer so the maximum strain would be compressive.

Table 7.3 shows the glue-line properties for the different diaphragms, while Table 7.4 presents the results for the different diaphragms.

The maximum strain profile for 16Std900 shows that the maximum strain profile for the bottom fibers of each layer is an inverted mirror image of the

* Maximum strain refers to the absolute quantity in the discussion.

* The 8th layer is nearest to the neutral axis of the system. Maximum strain in this layer would be compressive and would be in the top fibers. The strain in the 8th zone is taken into consideration because the constant zone would lie within the vicinity of this layer.

TABLE 7.3.--Properties of gluelines in different diaphragms.

Diaphragm	Zone	Number of Gluelines in the Zone n	G psi	b inch	t inch	s psi	sayg psi	Amount of Glue Per Inch in ³ /in
16Std ₉₀₀	a)	5	75	0.75	0.0625	900	900	0.703125
	b >	5						
	c)	5						
Diaphragm I	a)	5	75	1.125	0.0625	1,350	900	0.703125
	b >	5						
	c)	5						
Diaphragm II	a)	5	75	1.00	0.0625	1,200	900	0.703125
	b >	5						
	c)	5						
Diaphragm III	a)	5	75	0.75	0.0625	900	700	0.546875
	b >	5						
	c)	5						
Diaphragm IV	a)	4	75	1.25	0.0625	1,500	800	0.625
	b >	7						
	c)	4						

TABLE 7.4.--The results of the analysis of the different diaphragms.

Diaphragm	Maximum Strain in the Topmost Layer in/in	Maximum Strain in the 8th Layer in/in	Maximum Deflection inch
16 ^{Std} 900	$0.294 * 10^{-3}$	$0.094 * 10^{-3}$	0.7744
Diaphragm I	$0.521 * 10^{-3}$	$0.154 * 10^{-3}$	1.192
Diaphragm II	$0.395 * 10^{-3}$	$0.1165 * 10^{-3}$	0.9273
Diaphragm III	$0.408 * 10^{-3}$	$0.1353 * 10^{-3}$	1.0823
16 ^{Std} 520**	$0.375 * 10^{-3}$	$0.14 * 10^{-3}$	1.192
Diaphragm IV	$0.70 * 10^{-3}$	$0.21 * 10^{-3}$	1.59
16 ^{Std} 390**	$0.4 * 10^{-3}$	$0.1875 * 10^{-3}$	1.59
Diaphragm V	$0.558 * 10^{-3}$	$0.1975 * 10^{-3}$	1.49

** Results interpreted from the curves (Fig. 7.3 and Fig. 7.4).

maximum strain profile for the top fibers of each layer (Fig. 7.7.1a). The same is observed in case of zone (a) and zone (c) of Diaphragm I (Fig. 7.7.1b). Therefore, the profiles for zone (a) and zone (c) each can be looked upon as the maximum strain profile for 6^{Std}_{1350} , while the profile for zone (b) would be that for 4^{Std}_0 . If 4^{Std}_0 alone is subjected to a uniformly distributed load of 100 lb per ft length, the maximum strain would be higher than 0.154×10^{-3} in/in. This implies that in Diaphragm I, most of the load is carried by zone (a) and the zone (c). Also, as stated above, the maximum strain profiles of zone (a) and zone (c) resemble the strain profile for any of the standard diaphragms.

Thus, zone (b) in Diaphragm I merely transfers the load from zone (a) to zone (c). This 'no-glue zone' is merely a link between the two glue zones. The maximum strains and deflection of Diaphragm I are higher than that of 16^{Std}_{900} (Table 7.4). The possible solution to reduce the deflection and the maximum strains is to glue the zone (b). This is done in Diaphragm II (Table 7.3). The zone (b) in Diaphragm II consists of glueline stiffness which is one-fourth the stiffness of gluelines for zone (a) or zone (c) (Fig. 7.4). This resulted in reduced values of strains (Fig. 7.7.2a) and deflections (Table 7.4). In case of Diaphragm I and Diaphragm II, the average stiffness of the glueline and the amount of glue is equal to that in case of 16^{Std}_{900} (Table 7.3). But, from mechanics point, it can be noted that the location of a glueline is also significant in determining the effectiveness of the glueline. The gluelines away from the neutral axis are more effective than the gluelines near the neutral axis. This fact is verified by the analysis of Diaphragm III (Fig. 7.6.2b). Diaphragm III has an average glueline stiffness of 700 psi and less amount of glue (Table 7.3). But, the deflection (Table 7.4) and the strains in Diaphragm III (Fig. 7.7.2b) are less than that in

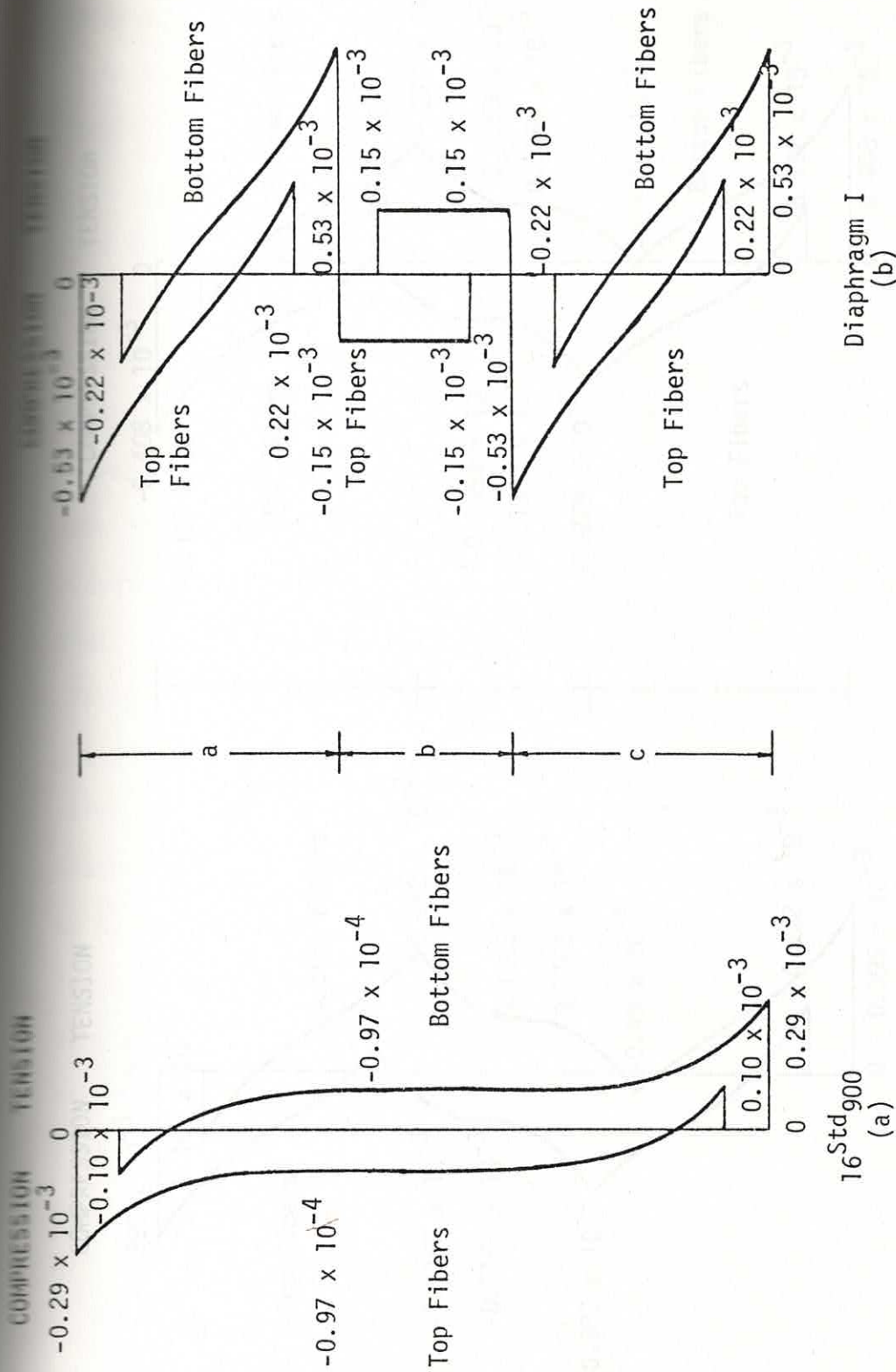


Fig. 7.7.1.--The profiles of the maximum strains in the top and bottom fibers of each layer in a diaphragm for 16Std900 and Diaphragm I.

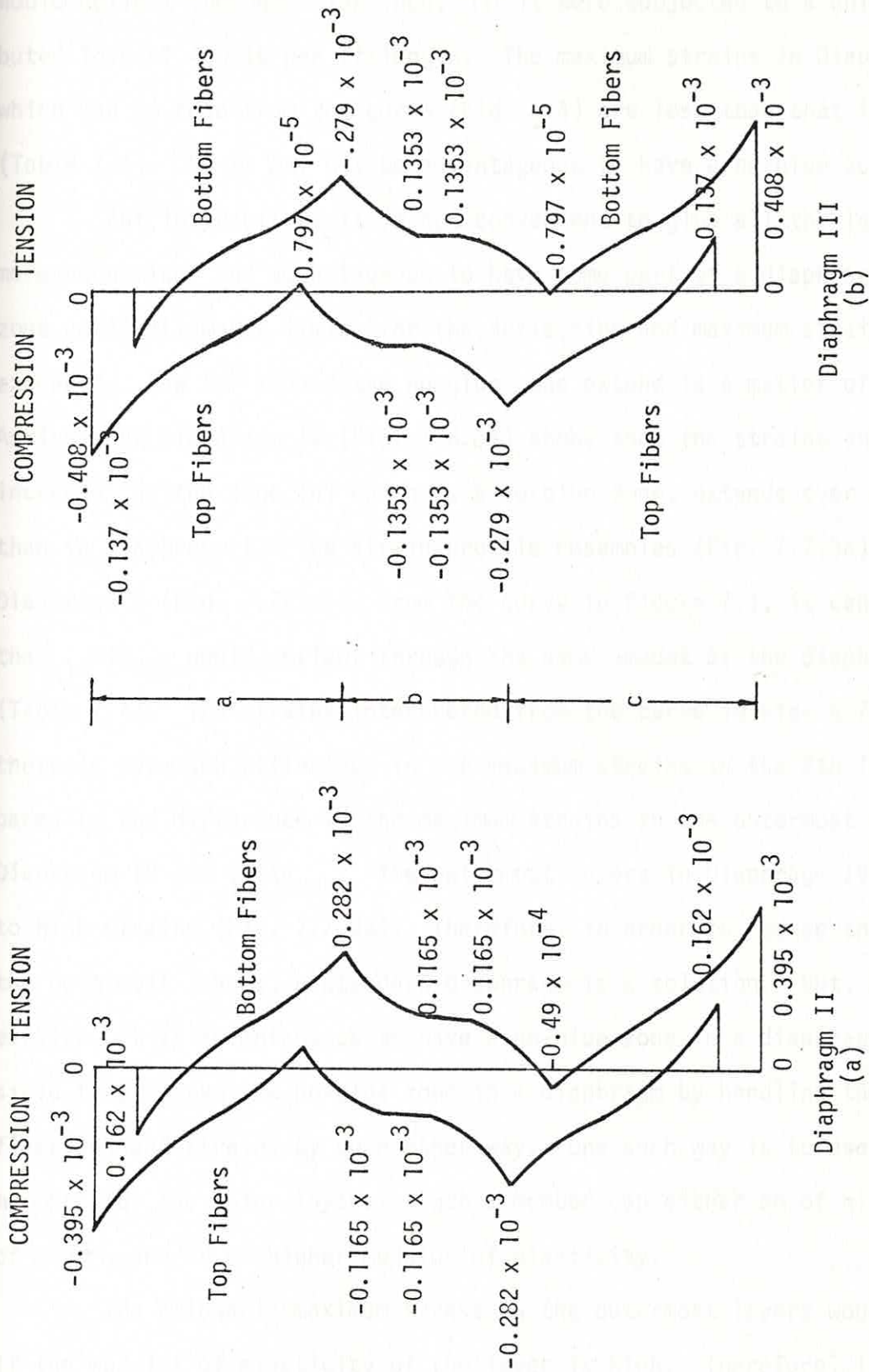


Fig. 7.7.2.--Profiles of the maximum strains in the top and bottom fibers of each layer in a diaphragm for Diaphragm II and Diaphragm III.

Diaphragm I (Fig. 7.7.1b). From Figure 7.1, it can be observed that 16^{Std}_{520} would deflect through 1.192 inch, if it were subjected to a uniformly distributed load of 400 lb per ft length. The maximum strains in Diaphragm III, which can be read from the curve (Fig. 7.4) are less than that in Diaphragm I (Table 7.4). So it may not be advantageous to have a no-glue zone.

But in practice, it is not convenient to glue all the layers. It is more economical and advantageous to have some part of a diaphragm as a no-glue zone until allowable limits for the deflection and maximum strains are not exceeded. How far should the no glue zone extend is a matter of judgment. Analysis of Diaphragm IV (Fig. 7.6.3a) shows that the strains and deflection increase as the zone (b) which is a no-glue zone, extends over a wider region than in Diaphragm I. The strain profile resembles (Fig. 7.7.3a) that of Diaphragm I (Fig. 7.7.1b). From the curve in Figure 7.1, it can be observed that 16^{Std}_{390} would deflect through the same amount as the Diaphragm IV would (Table 7.4). The strains interpreted from the curve in Figure 7.4 show that there is not much difference in the maximum strains in the 8th layer as compared to the difference in the maximum strains in the outermost layer of the Diaphragm IV and 16^{Std}_{390} . The outermost layers in Diaphragm IV are subjected to high strains (Fig. 7.7.3a). Therefore, in order to reduce the strains in the outermost layers, a standard diaphragm is a solution. But, as stated earlier, it is advantageous to have a no-glue zone in a diaphragm. It is possible to preserve the no-glue zone in a diaphragm by handling the high deflections and strains by some other way. One such way is to use 'heavier' members for the outer layers. Such a member can either be of higher moment of inertia and/or of higher modulus of elasticity.

The allowable maximum stress in the outermost layers would be high if the modulus of elasticity of the layer is high. Therefore, it is worthwhile

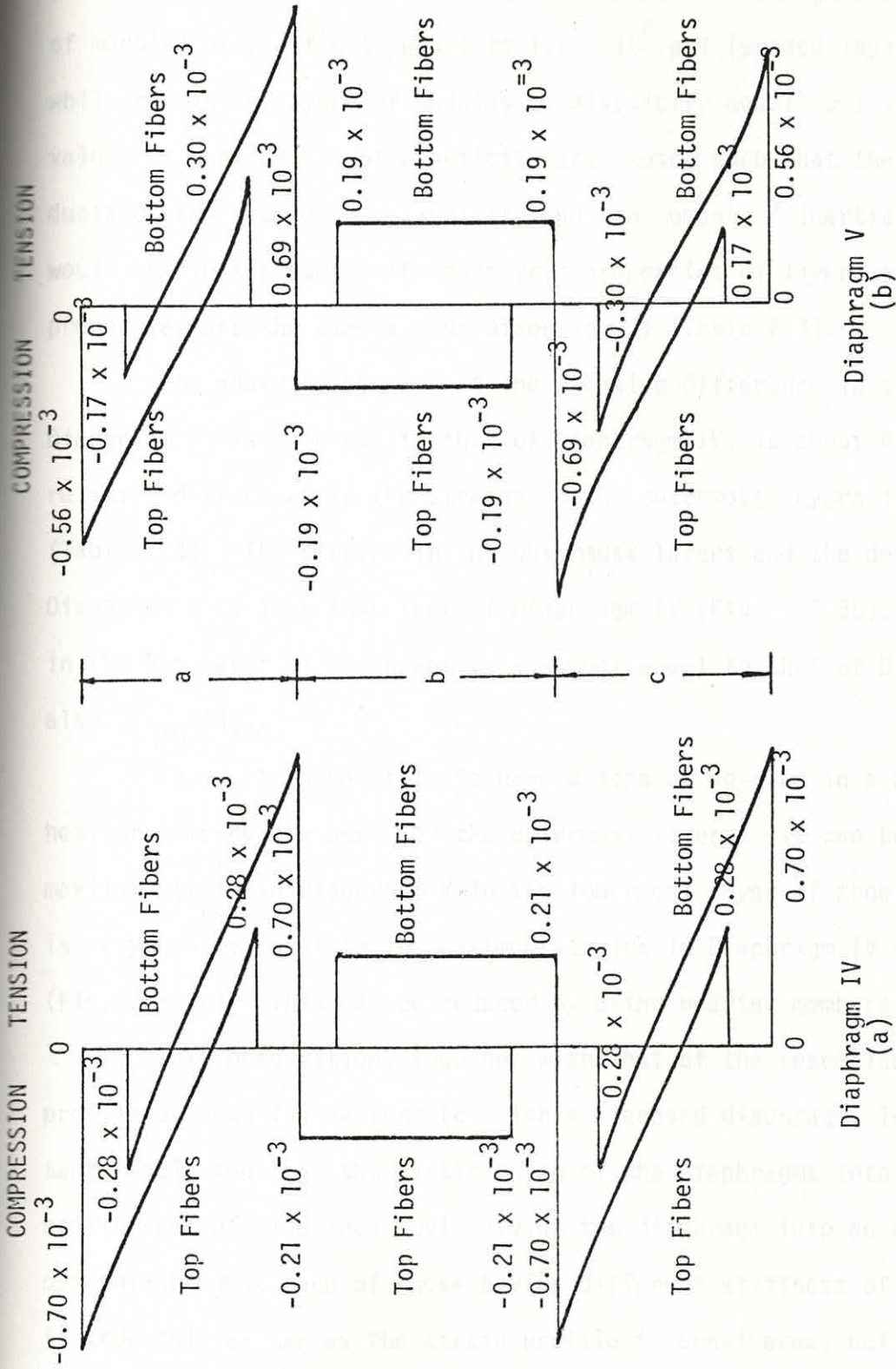


Fig. 7.7.3.--The profiles for Diaphragm IV and Diaphragm V of the maximum strains in the top and bottom fibers of each layer in a diaphragm.

Looking into one such diaphragm where outer layers have higher modulus of elasticity than that of the remaining layers. Diaphragm V has outermost layers of modulus of elasticity equal to 1.9×10^6 psi (shaded layers in Fig. 7.6.3b) while remaining layers of modulus of elasticity equal to 1.1×10^6 psi. The values of the modulus of elasticity are chosen such that the sum of the products of the modulus of elasticity and the moment of inertia for each layer would remain the same. The geometric properties of layers and the glue line properties are the same as for Diaphragm IV (Table 7.3).

The analysis shows that the relative difference in the deflection of Diaphragm V, as compared to that of Diaphragm IV, is about 6%, while the relative difference in the strains in the outermost layers is about 20% (Table 7.4). The strains in the outermost layers and the deflection of Diaphragm V is less than that of Diaphragm IV (Fig. 7.7.3b). But, the strains in the 8th layer of Diaphragm V is almost equal to that of Diaphragm IV and also of 16Std_{390} .

Thus, it is possible to have a zone of no-glue in a diaphragm if heavier members are used for the outermost layers. It can be noted that the maximum strain in Diaphragm V in the lowermost layer of zone (a) or zone (c) is very nearly equal to the maximum strains in Diaphragm IV at the same level (Fig. 7.7.3b). This can be reduced by using heavier members at that level.

This observation, together with that of the resemblance in the strain profile of zone (a) or zone (c) with a standard diaphragm, leads to an important conclusion that the partitioning of the diaphragms into zones of different stiffnesses of glue lines would divide the diaphragm into an assembly of standard diaphragms, each of these having different stiffness of glue lines. This is true only as far as the strain profile is considered, not if the magnitude

of the strain is considered. The load resisting action is the combined effort of this assembly of diaphragms. While introducing a no-glue zone, it is necessary to take care of the boundary members of the glue zones. This could be done by steadily decreasing the stiffnesses of gluelines towards the mid-depth. But, to account for the reversal of the load, it is necessary that the diaphragm be 'structurally symmetric.' It may be recalled that a diaphragm was said to be symmetric if the arrangement of the layers and the gluelines of different properties is symmetric about the mid-depth of the diaphragm.

The above discussions lead to certain useful conclusions which are presented in the next section.

7.6. Conclusions

1. A large number of layers can be used in a diaphragm, if the deflections are taken care of, by providing heavier members away from the mid-depth of the diaphragm.
2. All the layers in a diaphragm need not be glued at the interfaces of the adjacent layers. A diaphragm can be partitioned in different sets of gluelines of different stiffnesses. In such cases, a diaphragm can be looked upon as an assembly of a number of diaphragms with different stiffnesses for the gluelines. A well glued region of a diaphragm would be located away from the mid-depth of the diaphragm, while the region in the vicinity of the mid-depth may not be glued.
3. The high deflections and strain because of the above, can be taken care by 'reinforcing' the outermost layers. Reinforcing may be in form of steel plates fastened to the outermost layers or in form of steel members as the outermost layers, or larger wood members.

CHAPTER 8

CONTINUING STUDY

8.1. Experimental Study

The set of governing differential equations developed for an n layer system forms the basis for the analysis of composite action in lumber diaphragms in light of the number of courses, the stiffnesses of the gluelines, and their arrangement. Finite difference method, used to solve these equations, may not be the best mathematical tool.

The theoretical analysis of a diaphragm is compared with the experimental work done by Johnson as discussed in Chapter 6. But, the diaphragm which was analyzed experimentally by Johnson [14] has some additional parameters which the theoretical analysis does not account for. Therefore, it would be useful to conduct some well instrumented experiments to verify strains and deflections of a diaphragm.

8.2. Theoretical Study

The analysis of the diaphragms leads to an important fact that there is a possibility of having a no-glue zone in a diaphragm. The spread of such a zone remains undecided. At the same time, a no-glue zone is to be associated with a zone having heavier members at the boundary. The possible combinations of such layers of different materials, if known, would be advantageous.

The relationship between the number of layers in a standard diaphragm of 60 ft x 20 ft size, and the maximum deflection for a particular stiffness

of the glue lines is developed in Chapter 7. If the relationship can be extended to any diaphragm, the maximum deflection can be predicted.

8.3. Application of this Study to Practical Design

The results so far obtained may contribute a good deal to the design and the construction of a diaphragm.

Guidelines for the partitioning of a diaphragm would be very valuable in practice for the design of diaphragms. This study can be a basis to establish such guidelines.

8.4. Summary

The development of a set of governing differential equations to account for the composite action in a multilayer diaphragm bonded with elastomeric adhesive, without openings and related study are the features of this work. Though all the diaphragms analyzed are subjected to a uniformly distributed load, the computer program--DAD--Different Analysis of Diaphragms developed in FORTRAN, can handle any general case of loading. The mathematical formulation is based upon finite difference method. The computer program is versatile in many respects (Appendix A).

The different curves for strains and deflections are presented. The possibility of partitioning of a diaphragm and having a no-glue zone in a diaphragm is concluded from the results. Experimental work with the results can give a new outlook to the design of diaphragms.

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APPENDIX A

USERS' GUIDE FOR DAD--DIFFERENCE ANALYSIS OF DIAPHRAGMS

1. To solve the problem, the user must first define the geometry of the diaphragm. This is done by specifying the length of the diaphragm, the thickness of the diaphragm, and the material properties of the diaphragm.
2. The user must then specify the boundary conditions for the diaphragm. This is done by specifying the displacement and the slope of the diaphragm at the ends of the diaphragm.
3. The user must then specify the load conditions for the diaphragm. This is done by specifying the magnitude and the location of the load on the diaphragm.

3.2. Control

The user must specify the control parameters for the program. These parameters are: the number of points to be used in the diaphragm, the number of points to be used in the load, and the number of points to be used in the boundary conditions.

1. The number of points to be used in the diaphragm is specified by the parameter N .
2. The number of points to be used in the load is specified by the parameter NL .
3. The number of points to be used in the boundary conditions is specified by the parameter NBC .

APPENDIX A

USERS' GUIDE FOR DAD--DIFFERENCE ANALYSIS OF DIAPHRAGMS

A computer program to facilitate the study of diaphragms was developed in FORTRAN. The User's Guide for this computer program DAD--Difference Analysis of Diaphragms is presented in this appendix. Illustrated problems are presented in Appendix B.

A.1. Salient Features of the Computer Program DAD

1. The program is based on the finite difference technique, employed to solve the set of governing of differential equations (Ref. Chapter 4).
2. The program computes the strains and deflections at various points along the length of the diaphragm taking into consideration various parameters discussed in Section A.2.
3. IMSL subroutines LEQT1F and LINV3F have been used in the program to solve linear algebraic equations.

A.2. Scope of the Program and Controlling Parameters--C.P.

The various control parameters--C.P., which are used in the program and the computations possible, are discussed in the following paragraphs:

1. Load Symmetry: C.P.: LOD

LOD = 1 if loading is symmetric about the midspan section.

LOD = 0 if loading is not symmetric about the midspan section.

If LOD is set equal to 1, all further computations are carried out for the half span of the diaphragm. If LOD is equal to 0, the M points within the interval [1, p-1] spaced at finite intervals 'h,' Figure 5.3 would be given as

$$M = \left(\frac{\text{span}}{h} - 1 \right)$$

Otherwise,

$$M = \frac{\text{span}}{2h}$$

When LOD is equal to 0, LINV3F is the IMSL subroutine used. For LOD equal to 1, LEQT1F is used.

2. Structural Symmetry: C.P.: ISYM

For the purpose of the present study, a diaphragm is said to be structurally symmetric if the material properties of layers and stiffnesses of the gluelines are symmetrically placed about the mid-depth of the diaphragm are identical. For example, if the diaphragm consists of n layers, the material properties of 1st and nth, 2nd and n-1th, 3rd and n-2th . . . etc., are identical. The same holds good for gluelines. The controlling parameter ISYM is used to specify structural symmetry. Thus,

ISYM = 1 if structural symmetry exists and is to be considered.

ISYM = 0 if structural symmetry does not exist, or the structural symmetry though exists, is not to be considered in computation.

When ISYM is set equal to 1, computations are reduced because magnitudes of the forces are symmetrical about mid-depth. Therefore, computations need be done only for the top half of the depth of the diaphragm.

The number of forces having different magnitudes in a diaphragm are given as follows:

$$NN = NL - 1 \quad \text{if structural symmetry does not exist, i.e., ISYM} = 0.$$

$$NN = \frac{NL}{2} \quad \text{if structural symmetry exists, i.e., ISYM} = 1.$$

where,

NL = the number of layers in a diaphragm.

3. Loading Condition: C.P.: MOM

The type of lateral in-plane load on the diaphragm can be accounted for by setting the controlling parameter MOM equal to either 1 or 0.

MOM = 0 if the load is uniformly distributed.

MOM = 1 if the load is not uniformly distributed.

If MOM is set equal to 1, the values of the bending moment due to the load at the M points spaced at finite intervals 'h' along the length of the diaphragm are to be read in as input.

If MOM is equal to 0, such values of the bending moments are generated by the program.

Only one loading condition is handled at a time.

4. Assignment of proper layer properties to different layers.

C.P.: IDATA, NSETS, NOSET, NOLAYS

Properties of layers can be assigned in two different forms by setting IDATA either equal to 1 or 0.

In case of cross-sections where the area and moment of inertia can be computed by the following relations, IDATA is set equal to 0.

$$\text{Area} = (\text{width}) * (\text{depth}) \quad (\text{A.1})$$

$$\text{Moment of Inertia} = \frac{(\text{width}) * (\text{depth})^3}{12} \quad (\text{A.2})$$

In such cases, the following 'Layer property set' is provided on each data card according to Format 200.

1. Width of the cross-section of the layer: B(K).
2. Depth of the cross-section of the layer: D(K)
3. Modulus of elasticity of the layer: E(K).

K would take values from 1 to NSETS, where,

NSETS = the number of data cards to read in layer property sets.

It must be mentioned that a separate data card is required for each different layer property set. A different layer property set is specified if one or more values of B(k), D(k), and E(k) are different from the preceding set.

When the values of the area and moment of inertia of cross-section of a layer cannot be given by the Equation (A.1) and Equation (A.2), respectively, IDATA is not equal to 1. This would enable the input data consisting of the layer property sets to be read according to the Format 201. The layer property set in this case consists of the following:

1. Area of the Cross-Section: AREA(K)
2. Moment of Inertia of the Cross-Section: B(K)
3. Depth of the Cross-Section: D(K)
4. Modulus of Elasticity of a Layer: E(K)

K would take values from 1 to NSETS.

NOSET and NOLAYS are the parameters to generate the array NOL(NL) which would assign a proper layer property set to each layer. The mechanism to generate NOL(NL) is self-explanatory.

NOSET = serial number of the layer property set in the input for the properties of layers.

NOLAYS = the number of layers associated with the NOSET.

5. Assignment of proper glueline properties: C.P.: LAY

The stiffness of a glueline is given as,

$$s = \frac{G \cdot b}{t}$$

where,

G is the shear modulus of the glue (force/in²).

b is the width of the glueline (inch).

t is the thickness of the glueline (inch).

G, B and t, as defined above, are the input values read as GG, TT, WB and then stored in the arrays G(NL-1), T(NL-1), WID(NL-1), respectively, for different layers. Only one data card specifying a set of values of GG, TT and WB is required, if the values are identical for a number of gluelines. The number of such gluelines is specified by LAY. The value of LAY is also read along with GG, TT and WB on the same data card.

When stiffness of a glueline is to be zero, GG or WB should be set to zero, but TT should not be set to zero.

6. Printout of shear forces and shear flows: C.P. NLA, SLE

Since finite difference method is employed to solve the set of governing differential equations, the values of a shear force between any two adjacent layers are evaluated at the intermediate M points, spaced at the finite intervals 'h'. The values of a shear force at the two ends are known to be zero from the boundary conditions. Therefore, the values of a shear force at the M intermediate points need to be printed.

The corresponding shear flow is maximum at the end points. So the shear flow is computed and printed out at all $(M+2)$ points within the interval $[0, p]$, if ISYM is equal to 0. Otherwise, the shear flow is computed and printed out at $(M+1)$ points, starting from the left support of the diaphragm.

NLA and SLE controls the printout as follows:

- If NLA = 0 NN values of shear forces and the corresponding shear flows are computed and printed out at sections specified by setting an appropriate value of SLE.
- If NLA = i the values of the shear force and the shear flow between the i th and $(i+1)$ th layer at specified sections are computed and printed out.
- If NLA = NL neither shear forces nor shear flows are printed out.
- If SLE = -H Values of the shear force and the corresponding values of shear force and the corresponding values of shear flows at all M and $M+2/M+1$ points, respectively, are printed out. H is the finite difference interval in inch.
- SLE = x where x is the distance in inch from the left support of the diaphragm. Values of shear forces and shear flows at the distance x would be computed and printed out.

7. Computations for Deflections: C.P.: Nil

Deflections of the diaphragm at 'M' points, spaced at finite intervals "h" would be computed and printed out.

8. Computations for Strains: C.P.: MNO, DS.

The values of maximum strains in the top and bottom fibers of all or at any specified section along the finite difference mesh of M points can be computed and printed out.

MNO and DS control the layer number and the section, respectively, desired for the computation and printout. Each of the controlling parameters works independently.

DS = the distance of a point from the left support (inch) at which computation of maximum strains need be done.

- DS = 0 In this case, the values of maximum strains at all the M points in the finite difference mesh along a glueline are computed and printed out.
- MNO = i Maximum strains in the ith layer from the top would be computed and printed out at the specified DS.
- MNO = 0 Maximum strains in all the alyers would be computed and printed out at the specified DS.

A.3. Brief Explanation of the Names of the Variables Used in the Computer Program

- AL Length of the diaphragm in inch.
- DS Distance of the section from the left support of the diaphragm where values of the maximum strains are required to be computed and printed out (inch).
- EI $\sum_{i=1}^{NL} (\text{Modulus of Elasticity}) * (\text{Moment of Inertia of a Layer})$
- GG Shear modulus of the glue (force/in²).
- H Finite difference interval (inch).
- ISYM Control parameters to specify structural symmetry.
- IDATA Control parameter to assign an appropriate layer property set to different layers.
- LAY Number of gluelines associated with a particular set of data GG, TT, and WB.
- LOD Control parameter to specify load symmetry.
- M Number of points in a finite difference mesh along a glueline.
 = Integer $\left(\frac{AL}{H} - 1 \right)$ if LOD = 0
 = Integer $\left(\frac{AL}{2} \cdot \frac{1}{H} \right)$ if LOD = 1
- MOM Control parameter to account for loads other than uniformly distributed loads.
- N Number of simultaneous linear equations obtained by employing finite difference technique.
 = M * NN

NN	Number of shear forces to be evaluated = $NL - 1$ if $ISYM = 0$ = $\frac{NL}{2}$ if $ISYM = 1$
NL	Number of layers in a diaphragm.
NSETS	Number of data cards to read in properties of layers, i.e., number of layer property sets.
NOSET	Serial number of the particular layer property set associated with the layer.
NOLAYS	Number of layers associated with NOSET.
NLA	Serial number of the shear force and corresponding shear flow to be printed out.
MNO	Layer number in which maximum strains in top and bottom fibers are to be computed.
Q	Uniformly distributed load per inch.
SLE	Distance of a section from the left support of the diaphragm where values of the shear forces and shear flows are to be computed and printed out (inch).
TT	Thickness of a glueline (inch).
WB	Width of a glueline (inch).

A.4. Array Names Used in Program

The array names used in the program are explained below.

A	Coefficient matrix in the system of linear algebraic equations. Minimum Dimension: $N * N$.
AREA	Values of area of cross-section corresponding to different layer property sets. Minimum Dimension: NSETS
B	Values of width of a section or moment of inertia of a section corresponding to different layer property sets. Minimum Dimension: NSETS
BM	Values of bending moments at different points in the finite difference mesh. Also, maximum strains in top fibers of different layers at a section. Minimum Dimension: Larger of M and NL

- D Values of depths corresponding to different layer property sets. Minimum Dimension: NSETS
- E Values of modulus of elasticity corresponding to different layer property sets. Minimum Dimension: NSETS
- F The column vector representing the known quantities. Also, shear forces at different points on the finite difference mesh along all the gluelines. Minimum Dimension: N
- G Values of shear modulus of the glue for each of the gluelines. Minimum Dimension: (NL-1)
- NOL Array to assign appropriate properties to each layer. Minimum Dimension: NL
- S Stiffnesses of gluelines. Minimum Dimension: NL-1
- T Thicknesses of gluelines. Minimum Dimension: NL-1
- WID Widths of gluelines. Minimum Dimension: NL-1
- WKAREA Necessary workspace to solve the system of linear algebraic equations used by the IMSL routines. Minimum Dimension: N
- Z Values of $\left(\frac{d_i + d_{i+1}}{2}\right)$ for every i . $i = 1, 2, 3 \dots, (NL-1)$
 where d_i = depth of the cross-section of the i th layer. Also, values of maximum strains in the bottom fibers of each layer at a section. Minimum Dimension: NL

APPENDIX B

THE COMPUTER PROGRAM--DAD AND ILLUSTRATIVE PROBLEMS

```

C
C PROGRAMM 'DAD'-DIFFERENCE ANALYSIS OF DIAPHRAGMS.
C DEPT. OF CIVIL & ENVIRONMENTAL ENGINEERING
C WASHINGTON STATE UNIVERSITY,1979.
C
C DIMENSION 8(2),D(2),E(2),Z(25),BM(25),AREA(2),NOL(16),WID(16),G(
116),T(16),S(16),A(160,160),F(160),WKAREA(180)
0001 1000 FORMAT(/,*****
0002 1*****
1*****
0003 PRINT 1000
C
C LINEAL MEASUREMENT IN INCH. UNITS OF FORCE MAY BE ADJUSTED
C
C READ 100,NL,NSETS,LOD,ISYM,G,AL,H
0004 100 FORMAT(2I4,2I1,3F10.4)

```

```

C
C
0006      PRINT 110,AL,NL
0007      110 FORMAT(/8X,'LENGTH OF DIAPHRAGM : ',F8.2,' INCH',12X,'NUMBER 0
          IF LAYERS : ',I4)
0008      PRINT 325,Q
0009      325 FORMAT(/8X,'LOAD PER INCH : ',E10.4)
C
C Q NEED NOT BE SPECIFIED IF MOM=1
C
0010      H=H/12,
C
C
0011      READ 101, IDATA, MOM
0012      101 FORMAT(2I1)
C
C

```

C NL IS NO. OF LAYER,L IS LENGTH,S IS EQ.TO GT/B IN PSI

C

0013 PRINT 224

0014 224 FORMAT(/8X,'PROPERTIES OF GUELINES',/8X,'-----

1--')

0015 PRINT 225

0016 225 FORMAT(/8X,'NO. OF GUELINE ',/8X,'RIGIDITY OF GLUE',/8X,'THICKNE

ISS ',/8X,' WIDTH ')

0017 PRINT 226

0018 226 FORMAT(/8X,' ',/8X,' PSI ',/8X,' INCH

1 ',/8X,' INCH ')

C

C ASSEMBLE PROPERTIES OF DIFFERENT GUELINES.

C

0019 KK=1

0020 II=0

0021 1 READ 114, LAY, GG,II,WB

0022 114 FORMAT(I5,3F10.5)

C

C A BLANK CARD IS NECESSARY TO TERMINATE THE DOLOOP.

C

0023 IF(LAY.EQ.0) GO TO 706

0024 PRINT 227,LAY,GG,TT,WB

0025 227 FORMAT(/13X,I4,19X,E10.4,12X,E10.4,5X,E8.3)

0026 II=II+LAY

0027 DO 2 I=KK,II

0028 G(I)=GG

0029 Y(I)=TT

0030 WID(I)=WB

0031 S(I)=GG*WB/TT

0032 2 CONTINUE

0033 KK=KK+LAY

0034 IF(LAY.NE.0) GO TO 1

0035 706 RE=Q*AL/(2.)

```

0036      DO 11 I=1,NL
0037          NOL(I)=0
0038      11 CONTINUE
C
C ASSIGN PROPER PROPERTY SET TO EACH LAYER.
C
0039          KK=0
0040          II=1
0041      12 READ 105, NOSET,NOLAYS
0042      105 FORMAT(3I5)
C
C A BLANK CARD IS NECESSARY TO TERMINATE THE DOLOOP.
C
0043          IF(NOSET.EQ.0) GO TO 707
0044          II=II+NOLAYS
0045          DO 14 I=KK,II
0046              NOL(I)=NOSET
0047      14 CONTINUE
0048          KK=KK+NOLAYS

```

```

0049      IF(NOSET.NE.0) GO TO 12
0050      707  IF(IDATA.EQ.0) GO TO 15
C
C ASSEMBLE PROPERTIES OF DIFFERENT PROPERTY SETS
C
C IF IDATA.NE.0
C
0051      READ 201,((AREA(K),B(K),D(K),E(K)),K=1,NSETS)
0052      201  FORMAT(F10.5,F10.6,F10.4,F15.4)
C
C
0053      GO TO 16
0054      15  DO 5 K=1,NSETS
C
C IF IDATA.EQ.0
C
0055      READ 200,(B(K),D(K),E(K))
0056      200  FORMAT(F10.4,F10.4,F15.4)
C

```

C

0057 AREA(K)=AREA(K)+8(K)*D(K)

0058 5 CONTINUE

0059 DO 7 I=1,NSETS

0060 B(I)=0.0

0061 7 CONTINUE

0062 DO 8 I=1,NSETS

0063 B(I)=B(I)+AREA(I)*(D(I)**2/12.)

0064 8 CONTINUE

0065 151 FORMAT(/8X,'PROPERTIES OF LAYERS',/8X,'-----')

1)

0066 16 PRINT 151

0067 PRINT 152

0068 152 FORMAT(/8X,'AREA',11X,'MOMENT OF INERTIA',5X,'Z=C1+C2',5X,'MODUL

IUS OF ELASTICITY')

0069 PRINT 153

```

0070      153  FORMAT(/8X,'SQ.IN',11X,'      IN**4      ',5X,' INCH ',5X,'
          1 PSI      ')
0071      PRINT 154,((AREA(K),B(K),D(K),E(K)),K=1,NSETS)
0072      154  FORMAT(/(8X,E10.4,10X,E10.4,9X,E8.2,10X,E10.4))
0073      EI=C.0
0074      DO 24 K=1,NL
0075      F(K)=0.0
0076      24  CONTINUE
0077      DO 25 K=1,NL
0078      F(K)=F(K)+14./((AREA(NOL(K))*E(NOL(K)))
0079      EI=EI+B(NOL(K))*E(NOL(K))
0080      25  CONTINUE
0081      1885  FORMAT(/6X,6(E13.6,4X)/)
0082      AL=AL/(H*12.)
0083      M=AL
0084      M=M-1
0085      NN=NL-1
0086      IF(L0D.EQ.1) M=(M+1)/2
0087      IF(1SYM.EQ.1) NN=(NN+1)/2

```

```

0088      DO 34 K=1,NN
0089          Z(K)=0.0
0090      34 CONTINUE
0091      DO 35 K=1,NN
0092          Z(K)=Z(K)+{(D(NOL(K)))+D(NOL(K+1)))/2.
0093      35 CONTINUE
0094          DO 62 I=1,M
0095              BM(I)=0.0
0096      62 CONTINUE
C
C ASSEMBLE VALUES OF BENDING MOMENTS AT DIFFERENT POINTS.
C
C IF MOM.EQ.0 VALUES OF BENDING MOMENTS ARE GENERATED.
C
0097          IF(MOM.EQ.0) GO TO 169
C
C IF MOM.NE.0 VALUES OF BENDING MOMENTS ARE READ IN.
C
0098          READ 400,(BM(I),I=1,M)

```

0099 400 FORMAT(10E8.2)

C

C

0100 PRINT 1202

0101 PRINT 1560,(BM(I),I=1,M)

0102 GO TO 167

0103 169 DO 63 I=1,M

0104 XI=I

0105 BM(I)=RE*XI*H*12.-Q*(XI*H*12.)**2/2.

0106 63 CONTINUE

0107 1202 FORMAT(/10X,'BENDING MOMENT')

C

C N=THE NUMBER OF LINEAR EQUATIONS GIVEN BY THE FINITE

C DIFFERENCE METHOD

C

0108 167 N=M*NN

C

```

0109          IA=160
C
C FORMATION OF A MATRIX -THE COEFFICIENT MATRIX IN THE LINEAR
C EQUATIONS BEGIN.
C
0110          DO 64 I=1,N
0111          DO 64 J=1,N
0112          64 A(I,J)=0.0
0113          DO 65 J=1,NN
0114          KI=M*(J-1)+1
0115          MI=M*J
0116          JA=NL-J-1
0117          TEMP1=0.0
0118          TEMP2=0.0
0119          TEMP3=0.0
0120          TEMP4=0.0
0121          TEMP1=(F(J)+F(J+1))
0122          TEMP2=144.*Z(J)*Z(J)/EI
0123          TEMP3=Z(J)*Z(J)*144./EI-F(J)

```

```
0124 TEMP4=TEMP1+TEMP2
0125 IF(1SYM.EQ.1) GO TO 173
0126 GO TO 171
0127 173 IF(JA.GT.J) TEMP4=TEMP4+TEMP2
0128 IF(JA.EQ.J) TEMP4=TEMP4+TEMP3
0129 171 DO 66 I=K1,M1
0130 A(I,I)=A(I,I)+TEMP4*S(J)+2./(H*H)
0131 IF(I.EQ.K1) GO TO 66
0132 KI=I-1
0133 A(I,KI)=A(I,KI)-1./(H*H)
0134 A(KI,I)=A(I,KI)
0135 IF(LOD.EQ.1) GO TO 77
0136 GO TO 66
0137 77 IF(I.EQ.M1) A(I,KI)=A(I,KI)-1./(H*H)
0138 66 CONTINUE
0139 DO 67 JJ=1,NN
0140 K2=M*(JJ-1)+1
0141 TEMP=C.0
0142 M2=M*JJ
```

```
0143 JB=NL-JJ
0144 IF (JJ.LE.J) GO TO 67
0145 TEMP=TEMP+(Z(J)*Z(JJ)*144.)/EI
0146 DO 802 KK=K2,M2
0147 II=KK-M*(JJ-J)
0148 INDEX=JJ-J
0149 TEMP1=0.0
0150 TEMP1=TEMP
0151 IF (INDEX.GT.1) GO TO 72
0152 A(KK,II)=A(KK,II)-F(JJ)+TEMP
0153 IF (ISYM.EQ.1) GO TO 179
0154 GO TO 181
0155 179 IF (JB.NE.JJ) A(KK,II)=A(KK,II)+TEMP1
0156 A(II, KK)=A(KK, II)
0157 IF (JB.EQ.JJ) A(KK,II)=A(KK,II)+A(KK,II)
0158 GO TO 68
0159 181 A(II, KK)=A(KK, II)
0160 GO TO 68
0161 72 A(KK, II)=A(KK, II)+TEMP
```

```

0162 IF(ISYM.EQ.1) GO TO 175
0163 GO TO 177
0164 175 IF(JB.NE.JJ) A(KK,II)=A(KK,II)+TEMP1
0165 A(II,KK)=A(KK,II)
0166 IF(JB.EQ.JJ) A(KK,II)=A(KK,II)+TEMP1
0167 GO TO 68
0168 177 A(II,KK)=A(KK,II)
0169 68 A(KK,II)=A(KK,II)*S(JJ)
0170 A(II,KK)=A(II,KK)*S(J)
0171 802 CONTINUE
0172 67 CONTINUE
0173 65 CONTINUE
C
C FORMATION OF A MATRIX IS OVER.
C
C 75 CONTINUE
C FORMATION OF THE R.H.S.-THE KNOWN SIDE IN THE LINEAR
C EQUATIONS BEGIN.
C

```

C

```

0174      DO 74 I=1,N
0175      74  F(I)=0.0
0176      DO 75 J=1,NN
0177      TEMP1=0.0
0178      K=M*(J-1)+1
0179      MM=M*J
0180      TEMP1=Z(J)*S(J)*144./EI
0181      DO 85 I=K,MM
0182      TEMP2=0.0
0183      II=0
0184      II=I-M*(J-1)
0185      TEMP2=BM(II)
0186      F(I)=F(I)+TEMP1*TEMP2
0187      85  CONTINUE
0188      75  CONTINUE

```

C

C FORMATION OF THE R.H.S. IS OVER.

C

```

C
C CALL IMSL SUBROUTINE TO SOLVE THE LINEAR EQUATIONS.
C
C
C
0189           IF(L00.EQ.0) GO TO 870
0190           640 CALL LEQTLF (A,I,N,IA,F,0,WKAREA,IER)
0191           GO TO 1015
0192           870 CALL LINV3F(A,F,2,N,IA,D1,D2,WKAREA,IER)
C
C
C THE VALUES OF THE SHEAR FORCES AT DIFFERENT POINTS ARE
C AVAILABLE.
C
C
C
0193           1015 PRINT 1000
0194           NZ=NL+1
C
C
C
C NLA IS THE NUMBER OF THE SHEAR FORCE OF WHICH VALUES

```

```

C
C
0195      ICRO=0
0196      431 READ 600,NLA,SLE
0197      600 FORMAT(I5,F10.2)
C
C
C A BLANK CARD IN NECESSARY AT THE END OF ALL VALUES OF NLA
C
C
0198      SA=SLE/(12.*H)
0199      NAS=SA
0200      IF(SLE.EQ.AL) NAS=NAS-1
0201      MAS=NAS+1
C AT THE DIFFERENT POINTS ARE DESIRED.
C
C NLA.EQ.0 IF VALUES OF ALL THE FORCES ARE DESIRED.
C SLE IS THE DISTANCE FROM LEFT SUPPORT TO THE SECTION AT WHICH VALUES OF
C SHEAR FORCE AND SHEAR FLOW ARE DESIRED.

```

```

0202 IF(NLA.EQ.NL) GO TO 505
0203 DO 46 J= 1,NN
0204 IF(ICRO.NE.0) GO TO 349
0205 IF(NLA.EQ.0) GO TO 469
0206 IF(J.NE.NLA) GO TO 46
0207 469 JM=0
0208 JN=0
0209 JQ=0
0210 K=0
0211 KK=0
0212 MM=M+2
0213 JM=1+M*(J-1)
0214 JN=M*J
0215 JQ=J+1
0216 IF(LOD.EQ.1) MM=MM-1
0217 DO 364 I=1,MM
0218 A(I,NZ)=0.0
0219 364 CONTINUE
0220 DO 634 I=1,MM
0221 IF(NAS.LI.0) GO TO 305

```

```

0222 IF(MAS.NE.I) GO TO 634
0223 305 K=I+M*(J-1)
0224 KK=K-2
0225 IF(I.EQ.1) A(I,NZ)=F(K)/H
0226 IF(I.EQ.2) A(I,NZ)=F(K)/(2.*H)
0227 IF(I.GT.M) GO TO 164
0228 IF(I.LE.2) GO TO 634
0229 416 A(I,NZ)=(F(K)-F(KK))/(2.*H)
0230 GO TO 634
0231 164 IF(LOD.EQ.1) GO TO 634
0232 IF(I.NE.MM) A(I,NZ)=(0.-F(KK))/(2.*H)
0233 IF(I.EQ.MM) A(I,NZ)=-F(KK)/H
0234 634 CONTINUE
0235 161 FORMAT(/8X,'LAYER NO.S',3X,2(I3,2X)/8X,'-----')
0236 IF(NAS.GE.0) GO TO 405
0237 PRINT 161,J,JQ
0238 PRINT 116
0239 116 FORMAT(/8X,'SHEAR FORCE BETWEEN THE ABOVE LAYERS')
0240 PRINT 1650,(F(I),I=JM,JN)

```

```

0241 1560 FORMAT(/(8X,5(E10.4,5X)))
0242 PRINT 1652
0243 1652 FORMAT(/8X,'SHEAR FLOW BETWEEN THE ABOVE LAYERS')
0244 PRINT 1560,(A(I,NZ),I=1,MM)
0245 1650 FORMAT(/23X,4(E10.4,5X)/(8X,5(E10.4,5X)))
0246 PRINT 1000
0247 IF(NAS.NE.0) GO TO 46
0248 405 LAS=NAS+M*(J-1)
0249 IAS=LAS+1
0250 SF=0.0
0251 IF(NAS.NE.0) SF=F(NAS)
0252 IF(SLE.EQ.AL) SF=0.0
0253 IF(SLE.EQ.AL) IAS=IAS+1
0254 PRINT 162,J,JQ,SLE
0255 162 FORMAT(/8X,'LAYER NO.S : ',3X,2(I3,1X),5X,'SECTION AT : ',F6.2/8X,
1'-----')
0256 PRINT 117,SF,A(IAS,NZ)
0257 117 FORMAT(/8X,'SHEAR FORCE : ',E10.4/8X,'SHEAR FLOW : ',E10.4)
0258 46 CONTINUE

```

```
0259      ICRO=ICRO+1
0260      IF(NLA.NE.0) GO TO 431
C
C COMPUTATIONS FOR DEFLECTIONS BEGIN
C
C
0261      505 DO 464 I=1,M
0262          A(I,NZ)=0.0
0263          464 CONTINUE
0264          DO 712 I=1,3
0265              DO 712 J=1,M
0266                  A(J,I)=0.0
0267              712 CONTINUE
0268              DO 281 I=1,M
0269                  TEMP=0.0
0270                  II=0
0271                  II=I
0272                  DO 283 J=1,NN
0273                      JJ=NL-J
```

```

0274      TEMPI=0.0
0275      TEMPI=TEMPI+F(II)*Z(J)
0276      IF(1SYM.EQ.0) GO TO 285
0277      IF(JJ.NE.J) TEMPI=2.*TEMPI
0278      285 II=II+M
0279      TEMP=TEMP+TEMPI
0280      283 CONTINUE
0281      A(I,NZ)=A(I,NZ)+(BM(I)-TEMP)/EI
0282      281 CONTINUE
0283      A(1,1)=2.
0284      A(1,2)=A(1,NZ)*H*H*144.
0285      DO 565 I=2,M
0286      C=1.
0287      II=I-1
0288      IF(LOD.EQ.0) GO TO 731
0289      IF(I.EQ.M) C=C*2.
0290      731 A(I,1)=2.-1./A(II,1)*C
0291      A(I,2)=144.*H*H*A(I,NZ)+C*A(II,2)/A(II,1)
0292      565 CONTINUE

```

```

0293      A(M,3)=A(M,2)/A(M,1)
0294      DO 575 I=1,M
0295          IJ=M-I
0296          IK=IJ+1
0297          IF(I.EQ.M) GO TO 575
0298          A(IJ,3)=(A(IJ,2)+A(IK,3))/A(IJ,1)
0299          575 CONTINUE
0300          PRINT 1000
0301          PRINT 1540
0302          1540 FORMAT(/8X,'DEFLECTION')
0303          PRINT 1560,(A(I,3),I=1,M)
0304          PRINT 1000
0305          C 47 (F15YN,EO.0) ALL,NL=FKR
0306          C DEFLECTIONS ARE COMPUTED.
0307          C 103 (F15YN,EO.0) ALL,NL=FKR
0308          C COMPUTATIONS FOR MAXIMUM STRAINS BEGIN.
0309          C 148 CONTINUE

```

C

```
0305      DO 38 I=1,M
0306      DO 38 J=1,NL
0307      A(I,J)=0.0
0308      38 CONTINUE
0309      DO 518 I=1,M
0310      DO 118 J=1,NN
0311      JA=NL-(J-1)
0312      K=I+M*(J-1)
0313      KK=K-M
0314      IF(J.EQ.1) GO TO 31
0315      IF(J.EQ.NN) GO TO 47
0316      A(I,J)=F(KK)-F(K)
0317      103 IF(ISYM.EQ.1) A(I,JA)=-A(I,J)
0318      GO TO 118
0319      31 A(I,J)=-F(K)
0320      47 IF(ISYM.EQ.0) A(I,NL)=F(K)
0321      IF(ISYM.EQ.1) GO TO 103
0322      118 CONTINUE
```



```

0331 IF (SE.EQ.0.) GO TO 145
0332 314 IF (SE.NE.DS) GO TO 577
0333 145 DO 570 II=1,NL
0334 BM(II)=0.0
0335 Z(II)=0.0
0336 570 CONTINUE
0337 1640 FORMAT(/8X,'RESULTANT STRAIN : POSITION',F6.1,2X,'INCHES',5X,'LAYER
      IR NO.',I5)
0338 DO 573 J=1,NL
0339 IF (MNO.EQ.0) GO TO 683
0340 IF (MNO.NE.J) GO TO 573
0341 683 BM(J)=A(I,J)/(E(NOL(J))*AREA(NOL(J)))-A(I,NZ)*D(NOL(J))/2.
0342 Z(J)=A(I,J)/(E(NOL(J))*AREA(NOL(J)))+A(I,NZ)*D(NOL(J))/2.
0343 573 CONTINUE
0344 IF (MNO.EQ.0) GO TO 714
0345 PRINT 1640,SE,MNO
0346 PRINT 1622,BM(MNO),Z(MNO)
0347 1622 FORMAT(/8X,'MAX STRAIN IN TOP FIBERS:',5X,E10.4//8X,'MAX STRAIN
      IN BOTTOM FIBERS:',2X,E10.4)

```

```
0348 PRINT 1000
0349 GO TO 577
0350 714 PRINT 1630,SE
0351 1630 FORMAT(/,8X,'RESULTANT STRAIN: POSITION',F6.1,2X,'INCH')
0352 125 FORMAT(/,8X,'MAXIMUM STRAIN IN THE TOP FIBERS')
0353 PRINT 125
0354 126 FORMAT(/,8X,'MAXIMUM STRAIN IN THE BOTTOM FIBERS')
0355 PRINT 1560,{BM(J),J=1,NL)
0356 PRINT 126
0357 PRINT 1560,{Z(I),I=1,NL)
0358 577 CONTINUE
0359 IKO=IKO+1
0360 IF(MNC.NE.0) GO TO 329
0361 IF(OS.NE.0.) GO TO 329
0362 STOP
0363 END
```

C INPUT DATA FOR DIAPHRAGM IN FIG (7.6.2.B)

16 111 33.3333 720.0 18.0

10

5 75.00 0.0625 0.75
5 75.00 0.0625 0.25
5 75.00 0.0625 0.75

1 16

22.5 421.875 15.0 120000.0

3 -18.0

1 90.0

/*

LENGTH OF DIAPHRAGM : 720.00 INCH NUMBER OF LAYERS : 16

LOAD PER INCH : 0.3333E 02

PROPERTIES OF GUELINES

NO. OF GUELINE	RIGIDITY OF GLUE	THICKNESS	WIDTH
	PSI	INCH	INCH

5	0.7500E 02	0.6250E-01	.750E 00
---	------------	------------	----------

5	0.7500E 02	0.6250E-01	.250E 00
5	0.7500E 02	0.6250E-01	.750E 00

PROPERTIES OF LAYERS

AREA MOMENT OF INERTIA Z=C1+C2 MODULUS OF ELASTICITY

SQ. IN IN**4 INCH PSI

0.2250E 02 0.4219E 03 0.15E 02 0.1200E 07

LAYER NO.S 3 4

SHEAR FORCE BETWEEN THE ABOVE LAYERS

0.1065E 04 0.2113E 04 0.3133E 04 0.4114E 04

0.5051E 04 0.5938E 04 0.6771E 04 0.7549E 04 0.8268E 04

0.8928E 04 0.9527E 04 0.1006E 05 0.1054E 05 0.1095E 05

0.1130E 05 0.1159E 05 0.1181E 05 0.1197E 05 0.1207E 05

0.1210E 05

SHEAR FLOW BETWEEN THE ABOVE LAYERS

0.7099E 03	0.7044E 03	0.6892E 03	0.6670E 03	0.6394E 03
0.6079E 03	0.5735E 03	0.5370E 03	0.4990E 03	0.4598E 03
0.4196E 03	0.3788E 03	0.3375E 03	0.2959E 03	0.2539E 03
0.2118E 03	0.1697E 03	0.1274E 03	0.8501E 02	0.4260E 02

0.0

DEFLECTION

0.9348E-01	0.1856E 00	0.2755E 00	0.3622E 00	0.4453E 00
0.5243E 00	0.5988E 00	0.6685E 00	0.7332E 00	0.7927E 00
0.8470E 00	0.8957E 00	0.9390E 00	0.9766E 00	0.1009E 01
0.1035E 01	0.1055E 01	0.1070E 01	0.1078E 01	0.1081E 01

RESULTANT STRAIN : POSITION 90.0 INCHES LAYER NO. 1

MAX STRAIN IN TOP FIBERS: -.2136E-03

MAX STRAIN IN BOTTOM FIBERS: -.2258E-04

C INPUT DATA FOR DIAPHRAGM IN FIG (7.6.3.B)

16 211 33.3333 720.0 18.0

10

4 75.00 0.0625 1.25

7 1.0 1.00 1.00

4 75.00 0.0625 1.25

2 1

1 14

2 1

22.5 421.875 15.0 1100000.0

22.5 421.875 15.0 1900000.0

16

1 360.0

11 360.0

/*

LENGTH OF DIAPHRAGM : 720.00 INCH NUMBER OF LAYERS : 16

LOAD PER INCH : 0.3333E 02

PROPERTIES OF GUELINES

NO. OF GUELINE	RIGIDITY OF GLUE	THICKNESS	WIDTH
	PSI	INCH	INCH
4	0.7500E 02	0.6250E-01	.125E 01
7	0.0	0.6250E-01	.500E 00

4 0.7500E 02 0.6250E-01 .125E 01

PROPERTIES OF LAYERS

AREA MOMENT OF INERTIA Z=CI+C2 MODULUS OF ELASTICITY

SQ.IN IN**4 INCH PSI

0.2250E 02 0.4219E 03 0.15E 02 0.1100E 07

0.2250E 02 0.4219E 03 0.15E 02 0.1900E 07

DEFLECTION

0.1255E 00	0.2495E 00	0.3708E 00	0.4884E 00	0.6017E 00
0.7100E 00	0.8126E 00	0.9092E 00	0.9994E 00	0.1083E 01
0.1159E 01	0.1228E 01	0.1289E 01	0.1343E 01	0.1388E 01
0.1426E 01	0.1455E 01	0.1476E 01	0.1489E 01	0.1493E 01

RESULTANT STRAIN : POSITION 360.0 INCHES LAYER NO. 1

MAX STRAIN IN TOP FIBERS: -.5589E-03

MAX STRAIN IN BOTTOM FIBERS: -.1694E-03

RESULTANT STRAIN : POSITION 360.0 INCHES LAYER NO. 11

MAX STRAIN IN TOP FIBERS: -.1947E-03

MAX STRAIN IN BOTTOM FIBERS: 0.1947E-03
